Problem 9.28  For the circuit shown in Fig. P9.28:

(a) Obtain an expression for \( H(\omega) = \frac{V_o}{V_i} \) in standard form.

(b) Generate spectral plots for the magnitude and phase of \( H(\omega) \), given that \( R = 10 \, \Omega \), \( L = 1 \, \text{mH} \), and \( C = 10 \, \mu\text{F} \).

(c) Determine the cutoff frequency \( \omega_c \) and the slope of the magnitude (in dB) when \( \omega/\omega_c \ll 1 \).

Solution:

(a) Voltage division yields

\[
H(\omega) = \frac{V_o}{V_i} = \frac{(R \parallel j\omega L)}{1/j\omega C + (R \parallel j\omega L)}
= -\frac{\omega^2 LC}{1 + j\omega L/R + j^2 \omega^2 LC}
= -\frac{(\omega/\omega_c)^2}{1 + j2\xi \omega/\omega_c + (j\omega/\omega_c)^2},
\]

with

\[
\omega_c = \frac{1}{\sqrt{LC}}, \quad \xi = \frac{\omega_c L}{2R}.
\]

(b) For \( R = 10 \, \Omega \), \( L = 1 \, \text{mH} \), and \( C = 10 \, \mu\text{F} \),

\[
\omega_c = 10^4 \, \text{rad/s}, \quad \xi = 0.5.
\]

Spectral plots of \( M \, [\text{dB}] \) and \( \phi(\omega) \) are shown in Figs. P9.28(a) and (b).
(c) For $\omega / \omega_c \ll 1$,

$$H(\omega) \simeq -\left(\frac{\omega}{\omega_c}\right)^2 \quad \implies \quad \text{slope} = +40 \text{ dB/decade}.$$
Problem 9.29  For the circuit shown in Fig. P9.29:

(a) Obtain an expression for $H(\omega) = \frac{V_o}{V_i}$ in standard form.

(b) Generate spectral plots for the magnitude and phase of $H(\omega)$, given that $R = 50 \ \Omega$ and $L = 2 \ \text{mH}$.

(c) Determine the cutoff frequency $\omega_c$ and the slope of the magnitude (in dB) when $\omega/\omega_c \ll 1$.

![Figure P9.29: Circuit for Problem 9.29.](image)

Solution:

(a) Voltage division yields

$$H(\omega) = \frac{V_o}{V_i} = \frac{(R \parallel 1/j\omega L)}{R + (R \parallel 1/j\omega L)}$$

$$= \frac{j\omega L/R}{1 + j2\omega L/R}$$

$$= \frac{1}{2} \left( \frac{j\omega/\omega_c}{1 + j\omega/\omega_c} \right),$$

with

$$\omega_c = \frac{R}{2L}.$$

(b) For $R = 50 \ \Omega$ and $L = 2 \ \text{mH}$,

$$\omega_c = \frac{50}{2 \times 2 \times 10^{-3}} = 1.25 \times 10^4 \ \text{rad/s}.$$
(c) For $\omega / \omega_k \ll 1$, 

$$H(\omega) \approx \frac{j\omega}{2\omega_k} \quad \implies \quad \text{slope} = +20 \text{ dB/decade}.$$
Problem 9.30  For the circuit shown in Fig. P9.30:

(a) Obtain an expression for \( H(\omega) = \frac{V_o}{V_i} \) in standard form.

(b) Generate spectral plots for the magnitude and phase of \( H(\omega) \), given that \( R = 50 \Omega \) and \( L = 2 \text{ mH} \).

![Figure P9.30: Circuit for Problem 9.30.](image)

Solution:

(a) Voltage division yields

\[
H(\omega) = \frac{V_o}{V_i} = \frac{R}{R + (R \parallel j\omega L)} = \frac{1 + j\omega L/R}{1 + j2\omega L/R} = 1 + j\omega/\omega_c
\]

with

\[
\omega_c_1 = \frac{R}{L} = 2.5 \times 10^4 \text{ rad/s},
\]

\[
\omega_c_2 = \frac{\omega_c_1}{2} = 1.25 \times 10^4 \text{ rad/s}.
\]

(b) Spectral plots of \( M \text{ [dB]} \) and \( \phi(\omega) \) are shown in Figs. P9.30(a) and (b).
Sections 9-6 and 9-7: Active Filters

Problem 9.31  For the op-amp circuit of Fig. P9.31:

(a) Obtain an expression for \( H(\omega) = \frac{V_o}{V_s} \) in standard form.

(b) Generate spectral plots for the magnitude and phase of \( H(\omega) \), given that \( R_1 = 1 \, \text{k}\Omega, \ R_2 = 4 \, \text{k}\Omega, \) and \( C = 1 \, \mu\text{F} \).

(c) What type of filter is it? What is its maximum gain?

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{circuit.png}
\caption{Circuit for Problem 9.31.}
\end{figure}

Solution:

(a)

\[ V_p = V_n = V_s \]
\[ I_1 + I_2 = 0, \]

or equivalently,

\[ \frac{V_s}{R_1 + j\omega C} + \frac{V_s - V_o}{R_2} = 0, \]

which leads to

\[ H(\omega) = \frac{V_o}{V_s} = \frac{1 + j\omega(R_1 + R_2)C}{1 + j\omega R_1 C} = \frac{1 + j\omega/\omega_1}{1 + j\omega/\omega_2}, \]

with

\[ \omega_1 = \frac{1}{(R_1 + R_2)C}, \quad \omega_2 = \frac{1}{R_1 C}. \]
(b) For $R_1 = 1 \, \text{k} \Omega$, $R_2 = 4 \, \text{k} \Omega$, and $C = 1 \, \mu \text{F}$,

$$\omega_{c_1} = 200 \, \text{rad/s}, \quad \omega_{c_2} = 1000 \, \text{rad/s}.$$  

Spectral plots are shown in Figs. 9.31(b) and (c).

(c) It is a high-pass filter. For $\omega \gg \omega_{c_2}$,

$$H(\omega) \approx \frac{\omega_{c_2}}{\omega_{c_1}} = 5.$$  

Hence, maximum gain $= 20 \log 5 \approx 14 \, \text{dB}$.
Problem 9.35  For the op-amp circuit of Fig. P9.35:
(a) Obtain an expression for \( H(\omega) = \frac{V_o}{V_s} \) in standard form.
(b) Generate spectral plots for the magnitude and phase of \( H(\omega) \), given that \( R_1 = 1 \, k\Omega \), \( R_2 = 20 \, \Omega \), \( C_1 = 5 \, \mu F \), and \( C_2 = 25 \, nF \).
(c) What type of filter is it? What is its maximum gain?

Solution: This is basically an inverting amplifier with a transfer function given by

\[
H(\omega) = \frac{V_o}{V_s} = -\frac{Z_f}{Z_s} = -\frac{(R_2 \parallel \frac{1}{j\omega C_2})}{R_1 + \frac{1}{j\omega C_1}} = -\frac{-j\omega R_2 C_1}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)} = -\frac{-j(\omega/\omega_c)}{(1 + j\omega/\omega_c_1)(1 + j\omega/\omega_c_2)}
\]

with

\[
\omega_c_1 = \frac{1}{R_2 C_1} = \frac{1}{20 \times 5 \times 10^{-6}} = 10^4 \text{ rad/s},
\]
\[
\omega_c_2 = \frac{1}{R_1 C_1} = \frac{1}{10^3 \times 5 \times 10^{-6}} = 200 \text{ rad/s},
\]
\[
\omega_c_3 = \frac{1}{R_2 C_2} = \frac{1}{20 \times 25 \times 10^{-9}} = 2 \times 10^6 \text{ rad/s}.
\]

(b) Spectral plots are shown in Figs. P9.35(a) and (b).
(c) This is a bandpass filter with corner frequencies of 200 rad/s and $10^6$ rad/s. In the intermediate range, its maximum gain is approximately

$$G \approx 20 \log \left( \frac{R_2}{R_1} \right) = 20 \log 0.02 = -34 \text{ dB}.$$