Introduction to Audio and Music Engineering

Lecture 3

- Strings: oscillations in time and space
- Modes of oscillation
Strings – Oscillations in time and space

\[ T = \text{tension} \]
\[ \mu = \frac{\text{mass}}{\text{length}} \]

Speed of wave on string:
\[ c \equiv \sqrt{\frac{T}{\mu}} \]

Solution is sinusoidal in time and space:
\[ y(x,t) = \sin(n\omega_0 t) \cdot \sin(n\pi \frac{x}{L}) \]
\[ y(0,t) = y(L,t) = 0 \]

\[ \omega_0 = \pi \frac{c}{L} \]
\[ f_0 = \frac{c}{2L} \]

You can “prove” this by plugging solution into original equation.

vertical forces cancel

mass \times acceleration = tension \times curvature

string is pulled downward
What is the speed of bending wave propagation for a string with mass density of 0.01 kg/m and a tension of 100 Nts?

a) 10,000 m/sec
b) 1,000 m/sec
c) 100 m/sec
d) 10 m/sec
## Typical guitar string mass per unit length and tension

640 mm length (25.2")

<table>
<thead>
<tr>
<th>String</th>
<th>Diameter (inches)</th>
<th>Frequency (Hz)</th>
<th>Mass/Length kg/m</th>
<th>Tension kg (lbs)</th>
<th>Wave Speed (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (1&lt;sup&gt;st&lt;/sup&gt;)</td>
<td>0.010</td>
<td>329.63</td>
<td>$0.401 \times 10^{-3}$</td>
<td>7.28 (16.0)</td>
<td>421.8</td>
</tr>
<tr>
<td>B (2&lt;sup&gt;nd&lt;/sup&gt;)</td>
<td>0.013</td>
<td>246.94</td>
<td>$0.708 \times 10^{-3}$</td>
<td>7.22 (15.9)</td>
<td>316.1</td>
</tr>
<tr>
<td>G (3&lt;sup&gt;rd&lt;/sup&gt;)</td>
<td>0.017</td>
<td>196.00</td>
<td>$1.140 \times 10^{-3}$</td>
<td>7.32 (16.1)</td>
<td>290.3</td>
</tr>
<tr>
<td>D (4&lt;sup&gt;th&lt;/sup&gt;)</td>
<td>0.026 (0.014 core)</td>
<td>146.82</td>
<td>$2.333 \times 10^{-3}$</td>
<td>8.41 (18.5)</td>
<td>188.0</td>
</tr>
<tr>
<td>A (5&lt;sup&gt;th&lt;/sup&gt;)</td>
<td>0.036 (0.015 core)</td>
<td>110.00</td>
<td>$4.466 \times 10^{-3}$</td>
<td>9.03 (19.9)</td>
<td>140.8</td>
</tr>
<tr>
<td>E (6&lt;sup&gt;th&lt;/sup&gt;)</td>
<td>0.046 (0.016 core)</td>
<td>82.41</td>
<td>$6.790 \times 10^{-3}$</td>
<td>7.71 (17.0)</td>
<td>105.5</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>103.4 lbs</td>
</tr>
</tbody>
</table>

A typical grand piano has 230 strings and nearly 30 tons of tension in total!
Strings – Oscillations in time and space

\[ \frac{d^2 y(x,t)}{dt^2} = c^2 \frac{d^2 y(x,t)}{dx^2} \]

\[ c^2 \equiv \frac{T}{\mu} \quad T = \text{tension} \]
\[ \mu = \text{mass/length} \]

Spatial part must be a sine wave to satisfy boundary constraints, \[ y(0,t) = y(L,t) = 0 \]

... but you can have \( n \) half cycles of the sine wave

Plug possible solution into original equation ...

\[ n^2 \omega_0^2 = c^2 n^2 \frac{\pi^2}{L^2} \]
\[ \omega_0 = \pi c \frac{1}{L} \]
\[ f_0 = \frac{c}{2L} \]

\[ f_n = n \frac{c}{2L} \]

So the solutions with more half-cycles of a sine wave have higher frequency!
Modes of oscillation

Fundamental “mode” of oscillation

Round trip distance = 2L
Round trip time \( T = \frac{2L}{c} \)

\[ f_0 \equiv \frac{1}{T} = \frac{c}{2L} \]

\[ \omega_0 \equiv 2\pi f = \pi \frac{c}{L} \]

From previous slide

Second mode

\[ \lambda = \frac{2L}{2} = L \]

\[ f_2 = \frac{2L}{2L} = 2f_0 \]
Higher modes

Spatial wavelength and frequency of the \( n \)’th mode

\[
\lambda_n = \frac{2L}{n} \quad f_n = n \frac{c}{2L} = nf_0
\]
Question

What is frequency of the fundamental mode of a string of length 1 meter with a mass density of 0.01 kg/m and a tension of 100 Nts?

a) 50 Hz
b) 100 Hz
c) 200 Hz
d) 500 Hz