Partial credit is more likely to be granted for neat, well-organized work. Please make judicious use of definitions to simplify your expressions.

1) Find expressions for the force of electrical origin \( f^e \) of the transducer shown below in terms of \((x,v_1,v_2)\) & also \((x,q_1,q_2)\). Obtain these expressions in the simplest way you can.

For modeling purposes, neglect fringing fields and assume the gap between the two electrode pairs is small but that mutual coupling is still negligible. Also, neglect the air gap between the electrodes and the dielectric slab.

2) The capacitance of the variable gap capacitive transducer shown below is approximated by:

\[
C(x) = \frac{\varepsilon_0 A}{x + \varepsilon x^2}
\]

where \( x(t) \) is the gap spacing, \( A \) is the area, and \( \varepsilon > 0 \) is the coefficient of a first-order correction intended to account for the effect of fringing.

a) Obtain an equation for the mechanical equilibrium under the condition of fixed electric charge \( q_o \), when a static, external force \( F_{\text{ext}} \) is applied to the movable plate. Assume the spring force is zero at \( x = g \). Do not solve for the spacing, \( x_o \).

b) Can this equilibrium be unstable? Explain the method used to answer this question.
3) Consider the capacitive sensor shown below. The transducer is biased by DC voltage $V_o$ as shown. Furthermore, the moving electrode has mass $m$ and is constrained to motion in the $x$ direction by a linear mechanical spring having spring constant $k$. Assume the spring force is zero at $x = 0$, in other words, the spring tends to restore the electrode to $x = 0$ for both + and - displacements. Ignore mechanical damping.

For reference, the overall N-form system matrix is:

$$\begin{bmatrix} f_{\text{ext}} \\ \phi \end{bmatrix} = \frac{1}{N} \begin{bmatrix} N^2 + (Z_m + Z_{\text{mo}})(Y_e + Y_{\text{eo}}) \\ Y_e + Y_{\text{eo}} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

a) Find expressions for all coefficients in the transducer matrix, that is: $Z_m'$, $N$, $Y_e$, $Z_{\text{mo}}$, and $Y_{\text{eo}}$. In these expression, you may assume the existence of an equilibrium at $x = x_0$.

b) Assuming that the perturbation motion of the electrode is known to be $x'(t) = x_{\text{ex}} \cos(\omega t)$, i.e., $\dot{x} = x_{\text{ex}}$, find an expression that relates the output voltage phasor $\hat{v}_{\text{out}}$ to the motional amplitude $x_{\text{ex}}$. Please leave your answer in terms of the appropriate coefficients used in the system matrix.

**HINT:** Take care with definitions for the output voltage & current.
4) The angle-dependent capacitance $C(q)$ of the rotational electrostatic transducer shown above is:

$$C(q) = \frac{\epsilon_0 D}{\ln \left( \frac{R_2}{R_1} \right)}$$

The movable (rotating) upper electrode is attached to a torsional spring with restoring torque $t_{spring} = -k(q - q_r)$, where $q_r$ is the rest position of the spring.

c) Find an expression for the equilibrium of this system at an applied voltage value of $v = V_o$. Do not attempt to solve for this equilibrium angle $q_e$.

d) If the voltage is fixed at $V_o$, can the system be unstable? If your answer is yes, what is the critical voltage value for instability?

e) If the charge is fixed at $q_o$, can the system be unstable? If your answer is yes, what is the critical charge value for instability?