In the early 1990's, Texas instruments developed a MEMS chip containing a rectangular array that consists of thousands of mirror elements that serve as individual pixels for computer projectors. The technology was first reported fifteen years ago: IEEE Spectrum, Nov 1993, pp. 27-31. To create and project the image, a narrow, collimated light beam (not shown) is scanned across the array using a set of high-speed rotating reflector mirrors. The mirrors are individually addressable (on/off control or grey scale) by application of DC voltage. The TI display provides bright, primary color image rendition for large format projector systems.

An idealized model (not to scale) for the rotational capacitive transducer used in the TI chip is shown in the sketch below.

![Idealized model diagram]

a) The capacitance of the address electrode for each mirror element may be approximated by the following expression in terms of the deflection angle $\theta$.

$$C(\theta) = \frac{D}{d} \ln \left( 1 + \frac{a}{d + w} \right)$$
Obtain a Taylor series expansion for the capacitance that is correct to second-order, that is, \( \hat{C} \).

b) Check your result for part (a) by showing that your Taylor series approximation conforms to the familiar parallel-plate capacitance expression in the \( \varphi = 0 \) limit.

c) Next, use lumped parameter electromechanics to obtain an expression for the torque of electrical origin \( \hat{T} \) in terms of voltage \( v \), angle \( \varphi \), and system parameters.

d) Use the Taylor series for \( C(\varphi) \) from part (a) to obtain expressions for the coefficients comprising the N-form transducer matrix of the electromechanical two-port network, that is:

\[
\begin{bmatrix}
\hat{\varphi} \\
\hat{v}
\end{bmatrix} =
\begin{bmatrix}
Z_m \\
N \hat{\varphi}
\end{bmatrix} 
+ 
\begin{bmatrix}
N^* \\
Y_e
\end{bmatrix} 
\begin{bmatrix}
\hat{\varphi} \\
\hat{v}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\hat{\varphi} \\
\hat{v}
\end{bmatrix} =
\begin{bmatrix}
Z_m \\
N \hat{\varphi}
\end{bmatrix} 
+ 
\begin{bmatrix}
N^* \\
Y_e
\end{bmatrix} \hat{v}
\]

e) Assume \( I \) = moment of inertia of the rotating mirror, \( k \) = the spring constant (Newtons/radian) with the spring at rest at \( \varphi = 0 \), and \( b \) = rotational damping coefficient. On the electrical side, the terminals are connected to a DC bias voltage \( V_o \) in series with perturbation voltage \( v_{in}(t) = v'\cos(\varphi t) \), where \( |v'| \ll |V_o| \). Find the system function relating small-signal deflection angle \( \hat{\varphi} \), the output variable, to small-signal input voltage \( v' \).

f) Identify any resonant frequency associated with the device.

Note: In the TI chip, the array elements are actually driven by DC, not sinusoidal AC; nevertheless, the all-important frequency response can be studied using the model you developed.