An Overview on Space-Time Block Coding

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Motivation

- Time-varying multipath fading makes reliable wireless transmission difficult.
- Diversity techniques (like receiver and transmitter diversity) can help mitigate the effects of multipath fading.
- Receivers are typically small and with limited processing powers. It is not feasible to have multiple antennas at the receiver.
- Other needs like Simple encoding and decoding algorithms, need for efficient open loop system etc.
- More economic and feasible approach is to have transmit diversity at base station.
A quick look at receiver diversity : MRC

Received Signal:
\[ r_0 = h_0 s_0 + n_0 \]
\[ r_1 = h_1 s_0 + n_1 \]

Receiver Combining Scheme:
\[ \tilde{s}_0 = h_0^* r_0 + h_1^* r_1 \]
\[ = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1^* n_1 \]

Detection Scheme:
Maximum Likelihood detection. \( s_i \) selected iff:
\[ d^2(r_0, h_0 s_i) + d^2(r_1, h_1 s_i) \leq d^2(r_0, h_0 s_k) + d^2(r_1, h_1 s_k) \quad \forall i \neq k \]
\[ \Rightarrow d^2(\tilde{s}_0, s_i) \leq d^2(\tilde{s}_0, s_k) \quad \forall i \neq k \]
Alamouti’s Scheme

• Uses Transmitter Diversity
• Transmitters handle the encoding and transmission sequence of symbols.
• Combining is done by the receiver
• Decision rule is maximum likelihood detection.
Alamouti’s Scheme

Encoding and transmission sequence:

Assuming that fading is constant across two consecutive symbols duration, we have:

\[ h_0(t) = h_0(t + T) = h_0 = \alpha_0 e^{i\theta_0} \]

\[ h_1(t) = h_1(t + T) = h_1 = \alpha_1 e^{i\theta_1} \]

The received signals are:

\[ r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0 \]

\[ r_1 = r(t + T) = -h_0^* s_1^* + h_1^* s_0^* + n_1 \]
Alamouti’s Scheme

The Combining Scheme:
\[ \tilde{s}_0 = h_0^* r_0 + h_1^* r_1 \]
\[ \tilde{s}_1 = h_1^* r_0 - h_0^* r_1 \]

Solving we get:
\[ \tilde{s}_0 = (\alpha_0^2 + \alpha_1^2) s_0 + h_0^* n_0 + h_1^* n_1 \]
\[ \tilde{s}_1 = (\alpha_0^2 + \alpha_1^2) s_1 - h_0^* n_1 + h_1^* n_0 \]

• The combined signals are detected by the maximum likelihood detector.
• The resulting combined signals are equivalent to that obtained from two-branch MRRC.
• The only difference is phase rotations on the noise components which do not degrade the effective SNR
Alamouti’s Scheme

• The diversity order of Alamouti's transmit diversity scheme with one receiver is equal to 2, which is same as that of two-branch MRRC.

• For higher order of diversity, multiple receive antennas at the remote units may be employed. In such cases, diversity order of 2M can be obtained with two transmit and M receive antennas.

• The encoding is done in both space and time and hence is a form of space-time coding.

[1]
Alamouti’s Scheme

BER performance of uncoded coherent BPSK for MRRC and Alamouti’s scheme in Rayleigh fading:

• Alamouti’s scheme with two transmitters and a single receiver is 3 dB worse than two-branch MRRC.
• The 3-dB penalty is incurred because each transmit antenna radiates half the energy in order to ensure the same total radiated power as with one transmit antenna.

[1]
Space Time Block Codes

- STBC are generalizations of Alamouti’s scheme to more than two transmit antennas.
- The theory of orthogonal designs to create analogs of Alamouti’s scheme, namely, space–time block codes, for more than two transmit antennas.
- We use $n \times n$ orthogonal designs, there are $n$ transmission antennas and $n$ time slots.
- Signals received by each antenna are linear combinations of the signals transmitted in each time slot.
- Decoding is Maximum Likelihood method.
**Space Time Block Codes**

**Real Orthogonal Designs for STBC:**

- A real orthogonal (square) design of size $n$ is an $n \times n$ orthogonal matrix with entries the indeterminates $\pm x_1, \pm x_2, \ldots, \pm x_n$.
- However, it can be proved* that an orthogonal design exists if and only if $n=2$, 4, or 8.
- There are $n$ transmission antennas and $n$ time slots.
- We consider orthogonal designs as they can provide maximum diversity.

Examples:

$n=2$

\[
\begin{pmatrix}
  x_1 & x_2 \\
  -x_2 & x_1
\end{pmatrix}
\]

$n=4$

\[
\begin{pmatrix}
  x_1 & x_2 & x_3 & x_4 \\
  -x_2 & x_1 & -x_4 & x_3 \\
  -x_3 & x_4 & x_1 & -x_2 \\
  -x_4 & -x_3 & x_2 & x_1
\end{pmatrix}
\]

$n=8$

\[
\begin{pmatrix}
  x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\
  -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\
  -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\
  -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & x_6 & -x_5 \\
  -x_5 & -x_6 & -x_7 & x_5 & x_1 & x_2 & x_3 & x_4 \\
  -x_6 & x_5 & -x_8 & x_7 & -x_2 & x_1 & -x_4 & x_3 \\
  -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\
  -x_8 & -x_7 & x_6 & x_4 & -x_3 & -x_2 & x_1
\end{pmatrix}
\]

[2], *Hurwitz-Radon Problem
Generalized Real Orthogonal Designs

• We may generalize the definition of linear processing orthogonal designs to create new and simple transmission schemes for any number of transmit antennas.
• This extend the design to non-square orthogonal matrices with any number of transmit antennas.

Definition: A generalized orthogonal design $G$ of size $n$ is a $p \times n$ matrix with entries $0, \pm x_1, \pm x_2, \ldots, \pm x_k$ such that $GG^T = D$ where $D$ is a diagonal matrix with diagonal $D_{ii}, i=1,2,\ldots,n$ of the form $(l_1^i x_1^2 + l_2^i x_2^2 + \ldots + l_k^i x_k^2)$ where the coefficients are strictly positive integers. The rate of $G$ is $R=k/p$. 

[2]
Space Time Block Codes

• Transmission with such generalized orthogonal design will have \( n \) transmitting antennas and \( p \) time slots.

• For \( m \) receiving antennas, the diversity will be \( nm \).

• We to construct high-rate linear processing orthogonal designs with full diversity order. We must, however, take the memory requirements into account. This means given \( R \) and \( n \), we must try to minimize \( p \).

• For a given \( R, n \), we define \( A(R, n) \) to be the minimum number \( p \) such that there exists a \( p \times n \) generalized orthogonal design with rate at least \( R \).

• A generalized orthogonal design attaining the value \( A(R, n) \) is called delay optimal.

[2]
Space Time Block Codes

- The following are some delay-optimal orthogonal designs with rate one, $A (1, n)$ for $n=3, 5$.

\[ G_3 = \begin{pmatrix}
    x_1 & x_2 & x_3 \\
    -x_2 & x_1 & -x_4 \\
    -x_3 & x_4 & x_1 \\
    -x_4 & -x_3 & x_2
\end{pmatrix} \]

$p=4, n=3$ and $k=4$.

\[ G_5 = \begin{pmatrix}
    x_1 & x_2 & x_3 & x_4 & x_5 \\
    -x_2 & x_1 & x_4 & -x_3 & x_6 \\
    -x_3 & -x_4 & x_1 & x_2 & x_7 \\
    -x_4 & x_3 & -x_2 & x_1 & x_8 \\
    -x_5 & -x_6 & -x_7 & x_5 & x_1 \\
    -x_6 & x_5 & -x_8 & x_7 & -x_2 \\
    -x_7 & x_8 & x_5 & -x_6 & -x_3 \\
    -x_8 & -x_7 & x_6 & x_5 & -x_4
\end{pmatrix} \]

$p=8, n=5$ and $k=8$.

$p=$ no. of time slots, $n=$ no. of Tx antennas, $k=$ no. of symbols

[2]
Complex Orthogonal Designs

– Now we consider transmit diversity schemes with complex orthogonal designs using complex constellation rather than only real symbols.

– We define a complex orthogonal design $\mathcal{O}_c$ of size $n$ as an orthogonal matrix with entries the indeterminates $\pm x_1, \pm x_2 \ldots \pm x_n$, their conjugates, or multiples of these indeterminates by $\pm i$ where $i = \sqrt{-1}$.

– This transmit diversity scheme achieves the full diversity of $nm$. An example of a $2 \times 2$ complex orthogonal design is given by Alamouti’s [1] scheme.

– **Existence of Complex Orthogonal Designs**: A complex orthogonal design of size $n$ (square) exists if and only if $n=2$. 

[2]
Space Time Block Codes

**Generalized Complex Orthogonal Designs**

- We can extend the theory to non square matrices of sizes $p \times n$.
- **Definition**: A generalized complex orthogonal design $G_c$ of size $n$ is a $p \times n$ matrix with entries $0, \pm x_1, \pm x_2, \ldots, \pm x_k, \pm x_1^*, \pm x_2^*, \ldots, \pm x_k^*$, or their product with $i$, such that $G_c G_c^T = D_c$ where $D_c$ is a diagonal matrix with diagonal entries $D_{ii}, i = 1, 2, \ldots, n$ of the form $(l_1^i x_1^2 + l_2^i x_2^2 + \ldots + l_k^i x_k^2)$ where the coefficients $l_1^i, l_2^i, \ldots, l_k^i$ are strictly positive integers. The rate of $G_c$ is $R = k/p$.
- Given $R$ and $n$ we minimize $p$.
Space Time Block Codes

• For a given $R, n$, we define $A_c(R, n)$ to be the minimum number $p$ such that there exists a $p \times n$ complex generalized orthogonal design with rate at least $R$.

• It can be proved that $R \leq 0.5$, we have $A_c(R, n) < \infty$. This means, there always exits rate $\frac{1}{2}$ complex generalized orthogonal designs.

$G^3_c = \begin{pmatrix}
    x_1 & x_2 & x_3 \\
    -x_2 & x_1 & x_4 \\
    -x_3 & -x_4 & x_1 \\
    -x_4 & -x_3 & -x_2 \\
    x_1^* & x_2^* & x_3^* \\
    -x_2^* & x_1^* & -x_4^* \\
    -x_3^* & x_4^* & x_1^* \\
    -x_4^* & -x_3^* & x_2^*
\end{pmatrix}$

• The adjacent matrix shows a rate $\frac{1}{2}$ code with $n=3$, $p=8$, $k=4$.

• These transmission schemes and their analogs for higher $n$ give full diversity, but lose half of their theoretical bandwidth efficiency.

• For $n=2$, Alamouti’s scheme gives a rate one design.

• There are also designs that give rates of $\frac{3}{4}$ and others.

[2]
Space Time Block Codes

Performance of STBC

The figure shows that significant gains can be achieved by increasing the number of transmit antennas with little decoding complexity.
Space Time Block Codes

**Capacity of STBC**

- Capacity of MIMO channel of given channel is given by:

\[
C = \log[\det(I + PHH^{*})] = \log(I + P \|H\|_f^2 + \ldots + P^R \det(HH^*)_R)
\]

- Capacity of STBC can be shown to be:

\[
C_{STBC} = \frac{K}{T} \log(I + P \|H\|_f^2)
\]

The difference in capacity is:

\[
\Delta C = \frac{(T - K)}{T} \log(I + P \|H\|_f^2) + \log(1 + \frac{S}{(I + P \|H\|_f^2)}) \quad \text{where} \quad S = P^2 \sum_{i_1 < i_2} \sigma_{i_1}^2 \sigma_{i_2}^2 + \ldots + P^R \prod_{i=1}^R \sigma_i^2
\]

and P=SNR

[3]
Space Time Block Codes

*Capacity of STBC*

The Capacity of STBC is optimal when:

- Code rate $K/T$ is one
- Channel rank is one i.e. $S=0$

If the number of receiving antennas is greater than 1, then the channel rank is always greater than one. Hence there is always loss in capacity where number of receiving antennas is greater than 1.

STBC is effective when code rate is 1 and channel rank is 1. Hence, there is only one code, Alamouti’s code (2 x 1), which doesn’t compromise with the code rate.

[3]
Space Time Block Codes

Applications:
- In UTRA (UMTS Terrestrial radio access) Release 7
- In IEEE 802.11n, IEEE 802.16-2004, IEEE 802.16e
- Satellite Communication
- Powerline Communication
Conclusion:

- Full Diversity
- Linear Processing at the Receiver
- Simple Encoding and Decoding
- Open Loop Transmit Diversity Technique
- No Bandwidth Expansion
  - Loss in Capacity
  - Delay
  - Channel Estimation Error

[3]
References


Thank You.
Any Questions?