Model for the Echolocation Problem

a) Transmission Waveform:

\[ u(t) = a(t)e^{j\Theta(t)}, \quad 0 \leq t \leq T \]

\[ \int_0^T |u(t)|^2 \, dt = 1, \quad \text{(Energy Normalization)} \]

b) Moving Point Target:

\[ r(t) = E \sqrt{s} u[s(t - \tau)] + n(t) \]

\( \tau = \text{time delay} \), \( s = \text{Doppler stretch factor} \)
\( n(t) = \text{white Gaussian noise} \)

c) Optimal Detector; (Matched Filter):

\[ h(t) = u^*[T - t], \quad 0 \leq t \leq T \]
Fig. 4.11 Correlation receiver.

\[ A(t) = s(t - T), \quad 0 \leq t \leq T, \]
\[ = 0, \quad \text{elsewhere.} \]

Fig. 4.12 Matched filter receiver.

Fig. 4.13 Receiver operating characteristic: known signal in additive white Gaussian noise.
Trajectory Diagram

\[ y(t) = \frac{R(t)}{C} \]

\[ \frac{R_0}{C} \]

\[ 45^\circ \]

\[ \beta > 0 \]

\[ \beta = 0 \]

\[ \beta < 0 \]

\[ t_0, 2t_0, t_c, t_s, t_e \]

\[ \beta = \frac{dR(t)}{dt} \]
d) **Matched Filter Output**; (Complex Envelope):

\[ |A_u(\tau, s)|^2 = \sqrt{s} \int_{-\infty}^{\infty} u(t) u^*[s(t - \tau)] \, dt \]

\( A_u(\tau, s) \) is the wideband ambiguity function for \( u(t) \)

e) **Properties of WBAF:**

\[ |A_u(\tau, s)|^2 \leq 1 = |A_u(0, 1)|^2 \]

(Maximum at the origin)

\[ |A_u(\tau, s)|^2 = 1 - [\lambda^2 \tau^2 + 2\gamma \tau(s - 1) + \eta^2 (s - 1)^2] \ldots \]

(Ellipses near the origin)

f) **Ellipse Parameters:**

\[ \lambda^2 = \int |u'|^2 \, dt - \int |u|^2 \, dt \] (rms bandwidth)

\[ \eta^2 = \int t^2 \, |u'|^2 \, dt - 1/4 \] (Generalized TB product)

\[ \gamma = \int t|u'|^2 \, dt - \text{Re}\{ \int u u'^* \, dt * \int u u'^* \, dt \} \]

(cross coupling coef)
g) **Doppler Invariant Optimization:**

\[
\min_{a, \theta} \eta^2 = J_1(a) + J_2(a, \theta)
\]

where

\[
\begin{align*}
J_1(a) &= \int t^2 a^2 dt - 1/4 \\
J_2(a, \theta) &= \int t^2 a^2 \theta^2 dt - [\int t a^2 \theta dt]^2
\end{align*}
\]

results:

1) \( J_2(a, \theta) \geq 0 \), for all \( a \), with equality iff

\[
\theta(t) = k \ln(t) + \theta_0
\]

\[
\omega(t) = \theta'(t) = k/t, \quad T(t) = 2\pi/\theta'(t) = k_1 t
\]

**Hyperbolic FM**    **Linear Period Modulation**

2) \( J_1(a) \geq 0 \) (Euler-Lagrange Formulation)*

\[
a(t) = k_a \frac{1}{\sqrt{t}} J_p(\beta t), \quad 0 \leq t \leq T
\]

*with energy and rms timewidth constraints*
Figure 2.7 Simulated *Myotis* Cruising Pulse Parameters
Figure 3. *Hypotia loxifera* Cruising Pulse (Pulse No. 3)

Figure 4. *Hypotia loxifera* Diving Pulse (Pulse No. 4)

Figure 5. *Hypotia loxifera* Geobit Pulse (Pulse No. 5)
Figure 2.8 Myotis Cruising Pulse Ambiguity Function