Optimum Relay Distances Across Different Modulations

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Goals

- Given a network with particular noise and path loss:
  - Given any two of modulation, node distance, and transmit energy, be able to give optimal third parameter
  - Be able to show the gain of using the optimum
  - Be able to show the sensitivity of the optimum
Last Semesters Equations

\[ E_b = \frac{\gamma}{d^n} \]

\[ P_{s,p} = (1 - P_{e,s})^{\frac{k}{b}} \]

\[ T = kP_{s,p} \]

\[ F_{sd} = \frac{k}{b} \left( \frac{\gamma + \rho}{T} \right) \]

\( Eb \) : Energy per bit
\( d \) : distance between nodes
\( Pe,s \) : Probability of error of symbol
\( Ps,p \) : Probability of success of packet
\( b \) : bits per symbol
\( k \) : bits per packet

Rho: Fixed energy
## Possible Symbol Error Functions

<table>
<thead>
<tr>
<th>Modulation</th>
<th>$P_{e,s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BPSK</strong></td>
<td>$Q(\sqrt{\frac{2E_b}{N_o}})$</td>
</tr>
<tr>
<td><strong>QPSK</strong></td>
<td>$2Q(\sqrt{\frac{2E_b}{N_o}})(1 - 0.5Q(\sqrt{\frac{2E_b}{N_o}}))$</td>
</tr>
<tr>
<td><strong>DPSK</strong></td>
<td>$0.5e^{\frac{-E_b}{N_o}}$</td>
</tr>
<tr>
<td><strong>MPSK</strong></td>
<td>$2Q(\sqrt{\frac{4\log_2(M)E_b}{N_o}}\sin(\frac{\pi}{M}))$</td>
</tr>
<tr>
<td><strong>FSK - C</strong></td>
<td>$Q(\sqrt{\frac{E_b}{N_o}})$</td>
</tr>
<tr>
<td><strong>FSK - NC</strong></td>
<td>$0.5e^{\frac{-E_b}{2N_o}}$</td>
</tr>
<tr>
<td><strong>MFSK</strong></td>
<td>?</td>
</tr>
<tr>
<td><strong>MQAM</strong></td>
<td>$1 - (1 - 2(1 - \frac{1}{\sqrt{M}}))Q(\sqrt{\frac{3\log_2(M)E_b}{(M-1)N_o}})^2$</td>
</tr>
</tbody>
</table>
Finding Optimum Distance

\[(d^*)^n = \frac{\rho}{\alpha(n + 1)E_b}\]

\[F_{sd} = \frac{k}{b} \left( \frac{E_b d^n + \rho}{k(1 - P_{e,s})^{\frac{k}{b}}} \right)\]

Solving the find the intersection of the two plots gives an optimum distance
Finding Optimum Distance

• Case 1: \( F_{sd} < F_{sr} \) \( \rightarrow \) Direct Transmission

• Case 2: \( F_{sd} > F_{sr} \) \( \rightarrow \) Hop

\[ F_{sd} = F_{sr} \] \( \rightarrow \) Optimum Link
QPSK Ratio between reajusting opt gamma and not

- BPSK
- QPSK
- 8-PSK
- 16-PSK
- 4-QAM
- 16-QAM

Distance (m)