Training Design for Information Rate Maximization over Amplify and Forward Relay Channels

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  System Model
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  - Optimal Power Allocation among Training and Data
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Background

One existing challenge for wireless communications system: **fading**

Advantages of Relay

▷ Relay can enhance coverage range
▷ Relay channel setup achieves diversity gains by sending additional copies of the signal through relays

Relaying Strategies

**Figure:** Amplify and Forward (AF)  **Figure:** Decode and Forward (DF)
Problem Statement

Goal: Maximizing mutual information lower bound for AF relay channel

Precondition:
1. Imperfect known CSI at destination (LMMSE used for channel estimation)
2. Overall power constraints at source and relay node

Optimization:
1. Optimal power allocation among training and data at source
2. Optimal power allocation among source and relay node
System Model: AF Relay Channel

Received signal at $R$

$y = h_1 s + n_1$

Received signal at $D$

$z = h_2 f y + n_2$

$= f g \underbrace{s + fh_2 n_1 + n_2}_{\text{overall channel}} + \underbrace{n_2}_{\text{noise}}$

How to learn AF relay channel: Training based channel estimation

- Received training signal at $D$
  
  $z_t = fgs_t + fh_2 n_{1t} + n_{2t}$

- Received data signals at $D$
  
  $z_d = fgs_d + fh_2 n_{1d} + n_{2d}$
Power Constraints

Power constraint at source: \( P_t + P_d = P_s \)

\[ P_d = \rho_d T_d = \alpha \rho T \]

\[ P_t = \rho_t T_t = (1 - \alpha) \rho T \]

Power constraint at relay:

\[ E\{f y^H f y\} = f^2 (P_s \sigma_{h_1}^2 + \sigma_{n_1}^2 T) \leq P_r \]

Amplifying gain:

\[ f \leq \sqrt{\frac{P_r}{P_s \sigma_{h_1}^2 + T \sigma_{n_1}^2}} \]

Total power constraint over source and relay node:

\[ P_s + P_r = P \]
Existing Tradeoff in Power Allocation

- Tradeoff of power allocation for source and relay
- Tradeoff of Power allocation for training and data

To solve:
1. what is the optimal power allocation given power constraint at source and relay node
2. what is the optimal power allocation ratio given power constraint at source node?
LMMSE estimator

Based on received signal during training transmission

Estimated overall AF channel

$$\hat{g} = E\{gz_t^*\}E^{-1}\{zz_t^*\}z_t$$

Variance of channel estimation error

$$\sigma_{\hat{g}}^2 = \sigma_g^2 - E\{gz_t^*\}E^{-1}\{zz_t^*\}E\{zg_t^*\}$$

$$= \frac{\sigma_g^2}{P_s\sigma_{h_1}^2 + T\sigma_{n_1}^2} \sigma_g^2 (1 - \alpha)\rho' + 1$$

MSE couldn’t show optimal power allocation
Performance Metric: Mutual information

\[ z_d = f \hat{g} s_d + f \tilde{g} s_d + fh_2 n_{1_d} + n_{2_d} \]

Mutual information lower bound:

\[ I(s_d, z_d | \hat{g}) \geq \frac{T - T_t}{T} E_{\hat{g}} \{ \log(1 + \frac{f^2 \hat{g} E\{s_d^H s_d\} \hat{g}^*}{\sigma_v^2}) \} \]

\[ = \frac{T - T_t}{T} E_{\hat{g}} \{ \log(1 + \rho_{\text{eff}} \tilde{g} \tilde{g}^*) \} \]

where

\[ \rho_{\text{eff}} = \frac{P - P_s}{P_s \sigma_{h_1}^2 + T \sigma_{n_1}^2} \sigma_g^2 \rho' T \]

\[ \frac{\alpha(1 - \alpha)}{-\alpha + \gamma'} \]

Note: maximizing \( \rho_{\text{eff}} \) is equivalent to maximizing mutual information lower bound \( I(s_d, z_d | \hat{g}) \)
Recall multivariable calculus: Second derivatives test

Suppose the second partial derivatives of $\rho_{\text{eff}}$ are continuous on a disk with center $(P_s, \alpha)$, and suppose that $\partial \rho_{\text{eff}}(P_s, \alpha)/\partial \alpha = 0$ and $\partial \rho_{\text{eff}}(P_s, \alpha)/\partial P_s = 0$ [that is, $(P_s, \alpha)$ is a critical point of $\rho_{\text{eff}}$]. Let

$$D = D(P_s, \alpha) = \frac{\partial^2 \rho_{\text{eff}}}{\partial^2 P_s} \frac{\partial^2 \rho_{\text{eff}}}{\partial^2 \alpha} - \left[ \frac{\partial^2 \rho_{\text{eff}}}{\partial P_s \partial \alpha} \right]^2$$

1. If $D > 0$ and $\frac{\partial^2 \rho_{\text{eff}}}{\partial^2 \alpha} > 0$, then $\rho_{\text{eff}}(\alpha, P_s)$ is a local minimum.
2. If $D > 0$ and $\frac{\partial^2 \rho_{\text{eff}}}{\partial^2 \alpha} < 0$, then $\rho_{\text{eff}}(\alpha, P_s)$ is a local maximum.
3. If $D < 0$, then $\rho_{\text{eff}}(\alpha, P_s)$ is not a local maximum or minimum.

To find the critical points we need to solve the two first order derivatives jointly:

$$\begin{align*}
\frac{\partial \rho_{\text{eff}}}{\partial P_s} &= 0 \\
\frac{\partial \rho_{\text{eff}}}{\partial \alpha} &= 0
\end{align*}$$

Boundary of $\rho_{\text{eff}}(P_s, \alpha)$: $P_s \in [0, P]$ and $\alpha \in [0, 1]$ 

$$\rho_{\text{eff}}(0, \alpha) = 0 \quad \rho_{\text{eff}}(P, \alpha) = 0 \quad \rho_{\text{eff}}(P_s, 0) = 0 \quad \rho_{\text{eff}}(P_s, 1) = 0$$
Optimal Power Allocation

Maximizing $\rho_{\text{eff}}$ over $\alpha$, for a given $P_s$ yields:

$$\hat{\alpha} = \arg \min_{0 < \alpha < 1} \frac{\alpha(1 - \alpha)}{\alpha - \gamma'} = \gamma' - \sqrt{\gamma'(\gamma' - 1)}$$

in which,

$$\gamma' = \frac{P - P_s}{P_s \sigma_{h1}^2 + T \sigma_{n1}^2} \sigma_{h2}^2 + \sigma_{n2}^2 + \frac{P - P_s}{P_s \sigma_{h1}^2 + T \sigma_{n1}^2} \sigma_g^2 P_s$$

we note that $\hat{\alpha} \in (0.5, 1)$
Optimal Power Allocation

After replacing $\hat{\alpha}$ into $\frac{\partial \rho_{\text{eff}}}{\partial P_s} = 0$

$$aP_s^4 + bP_s^3 + cP_s^2 + dP_s + e = 0$$

where

$$a = k_0k_2 - k_0k_1,$$
$$b = 3Pk_0k_1 - 2k_1^2 + 4k_1k_2 + 2k_0k_3 - Pk_0k_2 - 2k_2^2,$$
$$c = -3P^2k_0k_1 + 6Pk_1^2 - 6Pk_1k_2 + 6Pk_1k_3 - 3Pk_0k_3 - 6k_2k_3,$$
$$d = P^3k_0k_1 - 6P^2k_1^2 + 2P^2k_1k_2 - 10Pk_1k_3 + P^2k_0k_3 + 2Pk_2k_3 - 4k_3^2,$$
$$e = 2P^3k_1^2 + 4P^2k_1k_3 + 2Pk_3^2$$

where

$$k_0 = \sigma_g^2(1 - \frac{T_d - 1}{T_d})$$
$$k_1 = \sigma_{h_2}^2\sigma_{n_1}^2$$
$$k_2 = \sigma_{h_1}^2\sigma_{n_2}^2$$
$$k_3 = T\sigma_{n_1}^2\sigma_{n_2}^2$$
Discussion

1. if $\sigma_{h_2}^2 \sigma_{n_1}^2 = \sigma_{h_1}^2 \sigma_{n_2}^2$

   $P_{s_1} = P/2 \star$

   $P_{s_2} = f_1(P, T, T_d, \sigma_{h_2}^2, \sigma_{n_1}^2, \sigma_{n_2}^2, \sigma_g)$

   $P_{s_3} = f_2(P, T, T_d, \sigma_{h_2}^2, \sigma_{n_1}^2, \sigma_{n_2}^2, \sigma_g)$

2. if $\sigma_{h_2}^2 \sigma_{n_1}^2 \neq \sigma_{h_1}^2 \sigma_{n_2}^2$

   $P_{s_1} = f_1(P, T, \sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_{n_1}^2, \sigma_{n_2}^2)$

   $P_{s_2} = f_2(P, T, \sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_{n_1}^2, \sigma_{n_2}^2) \star$

   $P_{s_3} = f_3(P, T, T_d, \sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_{n_1}^2, \sigma_{n_2}^2, \sigma_g)$

   $P_{s_4} = f_4(P, T, T_d, \sigma_{h_1}^2, \sigma_{h_2}^2, \sigma_{n_1}^2, \sigma_{n_2}^2, \sigma_g)$

Thus from optimal $P_s$, we can achieve optimal $\alpha$
Simulation: $\sigma^2_{h1} \sigma^2_{n2} = \sigma^2_{h2} \sigma^2_{n1}$
Simulation: \( \sigma_{h1}^2 \sigma_{n2}^2 \neq \sigma_{h2}^2 \sigma_{n1}^2 \)
Varying Block Length

![Graph showing mutual information over training power fraction for different block lengths T=20 and T=100.](image-url)
Conclusions

1. We used channel estimation for the overall AF relay channel
2. Mutual information Lower bound derived
3. Through maximizing mutual information lower bound, we achieved
   3.1 Optimal power allocation among source and relay node subject to overall power constraint
   3.2 Optimal power allocation among training and data at source node
Thank you and any Questions?