The model captures both topological ($A'$) and external influences ($X$).

**Problem Statement:**
Given $Y[t]_{1:T}$ and $X$ adhering to (1), track the underlying network topology ($A'_{1:T}$) and the effect of external influences ($B'_{1:T}$).

**Sparse exponentially-weighted least squares estimator**

Assuming the network topology changes slowly and has sparse edge connectivity, the estimator

$$
(A', B') = \arg \min_{A,B} \sum_{t=1}^{T} \frac{1}{2} \sum_{i,j} (Y[t][i] - A[i,j]X[j] - B[i])^2 + \lambda |A|,
$$

where $\beta \in (0,1]$, $\lambda > 0$, and $|A| = \sum \sum |a|$, tracks $A'$ and $B'$ where $\beta \in (0,1]$, $\lambda > 0$, and $|A| = \sum \sum |a|$.

**Benefits:**
1. Causal interactions among nodes (topological influences)
2. Time-varying interactions in psychometrics, social sciences, and gene regulation [Goldberger72][Cai13]
3. Time-invariant SEM for gene network inference [Cai13]

**Goal:**
Let $C$ cascades propagate during time interval $t \in [0,T]$, and exhibit slow variations.

**Related work:**
1. Maximum likelihood estimation (MLE) for static network inference [Rodriguez14]
2. MLE-based stochastic gradient descent for dynamic network inference [Rodriguez13]
3. Time-invariant SEM for gene network inference [Cai13]

**Contributions:**
1. Dynamic SEM for tracking time-varying networks
2. Accounting for external (non-topological) influences in cascades

**Conclusions and Related Work**

- Node infection times depend on:
  1. Causal interactions among nodes (topological influences)
  2. Susceptibility to cascades (external influences)
- Structural equation models (SEM) provide a general statistical framework for capturing causal interactions in psychometrics, social sciences, and gene regulation [Goldberger72][Cai13]

**Synthetic dataset:**
Cascades data generated from $Y[t] = (X - A'[t]B'[t]X) + e[t]$, where $e[t] \sim \mathcal{N}(0, I)$. $A'[t]_{0,1} \sim \mathcal{N}(0, 100)$. $X = 100$, $C = 150$, $T = 1, \ldots, 1000$. Edge weights were varied as $i)$ $a'[t] \sim \text{Bernoulli}(0.5)$

**Real datasets:**
Popular "memes" on the web were tracked between March 2011 and February 2012 [Rodriguez13].

**Numerical Results**

**Proximal Gradient Algorithm**

PG iterations with equality constraints yield the (pseudo) real-time tracking algorithm

**Model and Problem Statement**

Consider a dynamic network of $N$ nodes, over which $C$ cascades propagate during $T$ time intervals.

The postulated dynamic SEM for infection time of node $i$ by cascade $c$ during time interval $t$ is

$$y_{i,c} = \sum_{j \neq i} a_{i,j}y_{j,c} + b_{c} + e_{i,c}.$$  

Let $Y'[t] = [y'[t]]$, $X = [x[t]]$, $B' = [b']$, and $B = \text{diag}(b_1, \ldots, b_N)$, collecting observations for $N$ nodes and $C$ contagions yields the dynamic matrix SEM

$$Y'[t] = A'[t]Y'[t] + B'X + B,$$  

$t = 1, \ldots, T.$

**Reference**


