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Pareto Optimal Primary Designs for Color Displays

Hao Xie, Carlos Eduardo Rodríguez-Pardo, Gaurav Sharma; ECE Dept. Univ. of Rochester; Rochester, NY, USA

Abstract

The choice of primaries for a color display involves tradeoffs between different desirable attributes such as a large color gamut, high spectral reproduction accuracy, minimal observer metamerism, and low power consumption. Optimization of individual attributes often drives primary choices in different directions. For example, expansion of color gamut favors narrow spectral bandwidth saturated primaries and minimization of observer metamerism favors broadband primaries. To characterize the tradeoffs between the different attributes in primary design for three primary and multiprimary displays, we propose a Pareto optimization framework for determining the complete range of available primary choices that optimally negotiate the tradeoffs between the metrics for the different attributes. Using results obtained in our proposed framework, we explore the impact of number of primaries, the relation between alternative design objectives, and the underlying primary spectral characteristics. The proposed strategy is more informative and comprehensive for primary design and primary selection, and can also be extended to co-optimize primary design and selection of control values to fully leverage the advantages of multiprimary displays.

Introduction

The design of the spectral power distribution (SPD) of the primaries plays a critical role in color display systems. The choice of primaries determines the display gamut, i.e., the range of colors that the display can reproduce. For example, to maximize the (chromaticity) gamut, the chromaticities of the RGB primaries in the recent Rec. 2020 standard [1] are defined such that correspond approximately to spectrally-monochromatic primaries with wavelengths of 630nm, 532nm, and 467nm, respectively. An alternative strategy for realizing a wider gamut is to utilize more than three primaries, which is done in multiprimary displays. For both three primary and multiprimary color displays, other display metrics like luminance or power consumption also depend on the primaries' characteristics, which means deliberate design is essential to construct practical display systems.

Prior work has considered several different metrics for evaluating a set of display primaries. Going beyond the simplistic consideration of only the 2D gamut chromaticity area, several metrics have also been defined in terms of the SPDs of the primaries, and designs that optimize these metrics have also been obtained. Specifically, primary designs have been optimized for maximizing coverage of a pre-specified target gamut volume [2] or absolute gamut volume in a perceptually uniform color space [3]. For wide gamut designs based on narrow spectral bandwidth primaries, observer metamerism is often a concern, and designs have been proposed to optimize spectral reproduction [4] and minimize observer metamerism [5, 6, 7]. These prior works, however, focus on single objective to be optimized, and the question of how to adequately trade off these metrics against each other has received little attention. Primary designs have been proposed to mitigate the tradeoff between color gamut volume and optical power [8, 9], and between color gamut area and observer metamerism [7]. Alternatively an importance-weighted optimization has also been proposed [10], where the overall objective function for display primary design is formulated as a weighted sum of several metrics. However, the assignment of importance weights is empirical, and may be hard to set a priori without knowing the nature of the inter-relations between the different metrics.

In this paper, we propose a multi-objective/Pareto optimization framework to investigate the optimal tradeoff relations among different display metrics. Instead of a single design optimizing a numerical metric quantifying a single display trait (or a weighted combination), the Pareto optimization framework characterizes the complete set of solutions for which none of the metrics quantifying the different traits can be improved upon without compromising performance of at least one of the other metrics. As a result, instead of optimizing a single trait while disregarding all others, the Pareto optimal solution space fully characterizes the range of available primary choices that optimally negotiate the tradeoffs between the different traits for color displays. Using results obtained in our proposed framework, we explore and quantify the impact of number of primaries and the relation between alternative design objectives. The proposed strategy is more informative and comprehensive for primary design and selection, and can also be extended to co-optimization of primary design and selection of control values to fully leverage the advantages of multiprimary displays.

This paper is organized as follows. The next section lays the mathematical foundation for our problem setting by introducing spectral models for the display system and for object colors and their inter-relations via colorimetric/spectral reproduction objectives. Metrics quantifying the display attributes of color gamut coverage, power consumption, and observer metamerism, are then defined and the multi-objective optimization problem is formulated in terms of these metrics. The following section describes our implementation of the Pareto optimization framework using a parameterized representation of the primaries for computational efficiency. Results obtained using the framework are presented next where the nature of the optimal tradeoff relations and the underlying spectral properties of the primaries are discussed. As summary of the conclusions and a discussion of the results forms the final section.

Preliminaries

In this section, we introduce a spectral model for the display, the spectral representation for the surface colors that the display will attempt to reproduce, and corresponding color representations taking into account observer variability. Finally, we discuss the control process for the display by which the control values for the primaries are determined.

Display and Object Spectral Distributions

For a display system with K ($K \geqslant 3$) primaries, we model the spectrum rendered by the display in terms of the K primary spectra and their specified control values as

$$S_d(\lambda) = \sum_{i=1}^{K} \alpha_i p_i(\lambda) + p_0(\lambda), \tag{1}$$

where $p_i(\lambda)$ is the spectrum of the ith ($1 \leq i \leq K$) primary, α_i, $0 \leq \alpha_i \leq 1$, is the corresponding control value, and $p_0(\lambda)$ is the display black spectrum emitted when all primary control values are set to 0. The display black spectrum $p_0(\lambda)$ arises from the combination of reflected ambient light as well as from "leakage" light emitted by the display. We will refer to $p_0(\lambda)$ as flare regardless of its source. Note that $S_d(\lambda)$ is a function of the control values α_i, and thus in the cases when this dependency needs to be highlighted, we opt for the more explicit notation $S_d(\lambda, \alpha_1, \ldots, \alpha_K)$.

Object stimuli that the display is expected to emulate and reproduce, are modeled as the product of the illuminant spectrum and the object reflectance. That is, the spectrum for an object stimulus is given by $S_o(\lambda) = l(\lambda) r(\lambda)$, where $l(\lambda)$ is the illuminant SPD and $r(\lambda)$ is the spectral reflectance of the object.

Color Representation

Given the stimulus SPD, a color representation can be determined in terms of tristimulus values computed using the observers' cone sensitivities, or more commonly XYZ color matching functions (CMFs), $x(\lambda)$, $y(\lambda)$, and $z(\lambda)$, as

$$\begin{aligned} X &= \int x(\lambda) S(\lambda)\, \mathrm{d}\lambda \\ Y &= \int y(\lambda) S(\lambda)\, \mathrm{d}\lambda \\ Z &= \int z(\lambda) S(\lambda)\, \mathrm{d}\lambda \end{aligned} \tag{2}$$

where $S(\lambda)$ can be either the display output spectrum $S_d(\lambda)$ or the object spectrum $S_o(\lambda)$. Standardized versions of the CMFs denoted as $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, and $\bar{z}(\lambda)$ are defined by the CIE [11]. To account for the individual differences in cone sensitivities, i.e., the phenomena of observer metamerism, in this paper we also consider a population of people: specifically, a dataset of M observers characterized by their individual CMFs $x_j(\lambda)$, $y_j(\lambda)$, and $z_j(\lambda)$ ($1 \leq j \leq M$).

Display Color Control

The traditional strategy for display reproduction relies on finding a set of display control values α_i such that a colorimetric match between the display spectrum $S_d(\lambda)$ and a reference tristimulus is achieved for the standard observer[1]. For three primaries displays, this colorimetric matching has been the main objective for color reproduction, offering a unique triplet of control values for each reference tristimulus contained in the display gamut, which is defined as the range of tristimulus values attainable with feasible control values. Multiprimary displays provide further flexibility for colorimetric reproduction, as multiple sets of control values may be found to obtain display response with the same tristimulus representation [12]. This flexibility allows designers to set additional reproduction objectives that can be used to improve the display performance. For instance, obtaining the closest spectral reproduction is a complementary objective that helps to reduce observer metamerism [13, 7]. For simplicity of exposition and of computation, in our work, we establish spectral reproduction as the objective for display reproduction. Specifically, for (approximately) reproducing the object SPD $S_o(\lambda)$ the corresponding display control values α_i are determined by solving the constrained least squares problem,

$$\begin{aligned} \underset{\alpha_1,\ldots,\alpha_K}{\text{minimize}} \quad & \|S_d(\lambda, \alpha_1, \ldots, \alpha_K) - S_o(\lambda)\|_2^2 \\ \text{subject to} \quad & 0 \leq \alpha_i \leq 1, \text{ for } i = 1, \ldots, K \end{aligned} \tag{3}$$

We choose this approach for uniformity of our treatment across varying numbers of primaries and for computational simplification. At the cost of additional computation, our overall Pareto optimization methodology is more broadly applicable, for instance, for control values are determined to ensure colorimetric reproduction while preserving smooth variations in the presence of device variation [12] or minimizing an observer metamerism metric [13, 7].

Display White and Flare

The white stimulus plays a critical role in color reproduction, in particular, in determining the viewer's state of adaptation. Traditionally, the spectrum in (1) with all control values α_i set to unity, i.e., maximum is considered as the display white and the chromaticity of this spectrum is constrained to match a specified white chromaticity and the corresponding luminance then determines the display's dynamic range. However, having a predetermined specification of the white in this manner conflicts with the spectral reproduction objective that we use in this paper. We therefore define the white by the tristimulus $\{X_W, Y_W, Z_W\}$ of the illuminant $l(\lambda)$ SPD. This assumption has also been adopted in other recent work [7] considering observer metamerism for multiprimary displays.

The impact of the display flare, modeled in (1) by the term $p_0(\lambda)$, is determined by its magnitude relative to the white stimulus. We assume that the flare $p_0(\lambda)$ is modeled as a fraction κ of the sum of the primaries, i.e.,

$$p_0(\lambda) = \kappa \sum_{i=1}^{K} p_i(\lambda), \tag{4}$$

where the scaling factor κ is determined to set the flare luminance Y_0 to a fraction ζ of the white luminance Y_W. It can be readily seen that $\kappa = \zeta Y_W / (\sum_i Y_i)$ where Y_i denotes the luminance corresponding to the ith primary $p_i(\lambda)$. This modeling procedure allows us to represent a consistent level of flare as the primary spectra vary, while still providing a spectral representation for the flare, which is necessary for our spectral reproduction assumption.

Problem Formulation

For presenting concrete Pareto optimality formulations and designs, in this paper, we use three important display metrics

[1] For the conventional colorimetric reproduction setting, display primaries are often specified in terms of their tristimulus values without specifying a corresponding spectrum. The spectral model that we utilize here is advantageous when considering observer metamerism, spectral reproduction accuracy, and power consumption.

quantifying color gamut coverage, power consumption, and observer metamerism, respectively. The proposed Pareto framework can readily include additional metrics defined in terms of the specified display parameters.

Display Metrics

Color gamut coverage

The color gamut for a display system is defined as the set of all tristimulus values for the display spectra $S_d(\lambda)$ in Equation (1) as the control values α_i vary over their feasible ranges between 0 and 1. Because the tristimulus space is not perceptually uniform, alternative gamut representations in perceptual spaces are usually considered for display design. Specifically, we quantify the color gamut by the gamut area coverage in the $u'v'$ uniform chromaticity scale of the CIELUV color space [11]. While we have previously considered display design optimization to maximize 3D gamut volume in a perceptually uniform color space [3], here we consider the computationally simpler 2D chromaticity area metric for quantifying the color gamut. We note that luminance is in part implicitly considered in our model because the luminance of primaries impacts the chromaticity gamut area in the presence of the flare.

Figure 1 shows an example for a three primary display, illustrating the chromaticity gamut area coverage metric that we utilize. Note that due to the flare, the chromaticity gamut for the display is not the triangle formed by the primaries. To simplify computation, instead of the exact gamut area we quantify the gamut coverage in terms of the area $\mathscr{G}_{uv}$ of the convex hull formed by the channel chromaticities (incorporating flare), which is also shown in Fig. 1 for the corresponding sample design. Also included in Fig. 1 are the chromaticity coordinates of the Macbeth DC color checker reflecting the coverage of these in the gamut.

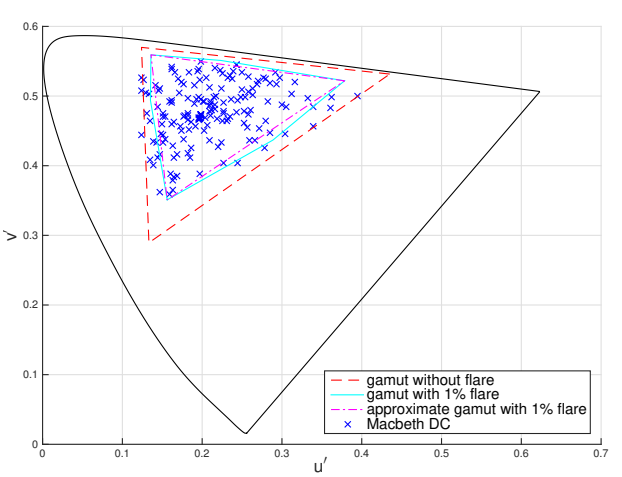

Figure 1. *Example illustrating the gamut coverage metric $\mathscr{G}_{uv}$ for a sample three primary display on the CIE u' v' chromaticity diagram. The red triangle vertices are the chromaticities of each primary. The flare, set to 1% ($\zeta = 0.01$) reduces the saturation of the primaries, moving them to the vertices of the smaller magenta triangle. The actual chromaticity gamut area coverage is the are enclosed in the cyan closed curve but for computational simplicity we approximate the gamut area coverage as $\mathscr{G}_{uv}$ the area enclosed by the magenta triangle. The blue cross marks correspond to 172 object colors from the Macbeth DC color checker under the CIE D65 illuminant (redundant neutral colors and glossy patches are excluded).*

Power consumption

The total optical power irradiated by a display correlates with the electrical power consumed by the system, thus the primary design with less optical power guarantees an efficient usage of the energy. The optical power of the ith primary is expressed as

$$\mathscr{P}_i = \int p_i(\lambda)\,\mathrm{d}\lambda\,, \tag{5}$$

Based on (1), the total optical power consumed when all primaries are at their maximum amplitudes is then given by

$$\mathscr{P}_W = \sum_{i=1}^{K} \mathscr{P}_i\,, \tag{6}$$

where we have ignored the optical power for the display flare. Note that for displays based on the modulation of a constant backlight, such as most of LCD displays, the power $\mathscr{P}_W$ represents the total power consumed any time the display is powered on, independent of what is displayed. In contrast, for displays that directly modulate their primaries, such as OLEDs and laser projectors, the optical power for reproducing a spectrum $S_d(\lambda)$ depends on the control values as

$$\mathscr{P}_{S_d} = \sum_{i=1}^{K} \alpha_i \mathscr{P}_i\,. \tag{7}$$

The control values determined in our case by Equation (3), can be alternatively optimized to minimize power consumption under a colorimetric match and the power consumption of displays that directly modulate their primaries is therefore determined by the strategy used to determine control values as well as by the distribution of colors reproduced. For simplicity, we use $\mathscr{P}_W$ as the power metric, which represents for all types of displays, an upper bound for the optical power.

Observer metamerism

Given that most displays are designed with three relatively broadband primaries, observer metamerism has not been widely considered in the process of display design. However, with the emergence of wide color gamut displays using narrow band primaries, the phenomenon of observer metamerism is gaining more attention [5]. The index of observer metamerism magnitude [14] $\mathscr{M}_O$ aligns well with our established objective of spectral reproduction and we therefore adopt it as the metric for quantification of observer metamerism instead of the more traditional CIE metamerism index [15].

To compute the observer metamerism index, consider a pool of M available observers, represented by their CMFs, $x_j(\lambda)$, $y_j(\lambda)$, and $z_j(\lambda)$, $j = 1,\ldots,M$, and a set of N object samples represented by the spectral power distribution $S_{o,n}(\lambda)$, $n = 1,\ldots,N$. For the jth observer, we denote by $\Delta E_{j,n}$ the Euclidean distance between the color representations, in CIELAB color space, of $S_{o,n}(\lambda)$ and the corresponding reproduced spectrum $S_{d,n}(\lambda)$ on the display. The display performance for this observer is quantified by the average error

$$\overline{\Delta E_j} = \frac{1}{N}\sum_{n=1}^{N} \Delta E_{j,n}. \tag{8}$$

The observer metamerism index $\mathscr{M}_O$ is defined as the worst reproduction error across all the observers, that is,

$$\mathscr{M}_O = \max_{j=1,\ldots,M} \overline{\Delta E_j}. \tag{9}$$

Multi-objective Optimization Problem

In this section, we formulate the problem of designing the primary spectra as a multi-objective optimization problem (MOOP) [16], aiming to examine the optimal tradeoffs between display metrics defined previously. More precisely, the optimal designs are a solution set for the following problem,

$$\begin{aligned} \underset{p_1(\lambda),p_2(\lambda),\ldots,p_K(\lambda)}{\text{maximize}} \quad & \{\mathscr{G}_{uv}, -\mathscr{M}_O, -\mathscr{P}_W\} \\ \text{subject to} \quad & p_i(\lambda) \geq 0,\ i = 1,2,\ldots,K \\ & p_0(\lambda) = \kappa \sum_{i=1}^{K} p_i(\lambda) \end{aligned} \tag{10}$$

where the constraints are set to guarantee physically realizable primaries with non-negative SPDs, and specify the amount of the flare with luminance ζY_W.

Implementation

To obtain and analyze Pareto optimal designs for our MOOP formulation in (10), we consider a parameterization of the display design space as follows.

Gaussian Primary Parameterization

Following Ref. [3], we parameterize $p_i(\lambda)$ as a Gaussian function,

$$p_i(\lambda) = \frac{\gamma_i}{\sqrt{2\pi\sigma_i^2}} exp\{-\frac{(\lambda-\mu_i)^2}{2\sigma_i^2}\}, \tag{11}$$

where parameter μ_i is the location of the peak wavelength, σ_i is the spectral width, and γ_i is the primary amplitude. A benefit of this model is that we can easily obtain the optical power consumed as [3]

$$\mathscr{P}_i = \int p_i(\lambda)\,\mathrm{d}\lambda = \gamma_i. \tag{12}$$

With this parameterization, the solution space of the optimization problem defined in equation (10) reduces to a $K \times 3$ dimensional space defined by the parameters $(\mu_i, \sigma_i, \gamma_i)$ for each of the K primaries.

Parameterized MOOP Formulation

With the parameterization and pruning, our final MOOP formulation becomes

$$\begin{aligned} \underset{\mathbf{t}}{\text{maximize}} \quad & \{\mathscr{G}_{uv}, -\mathscr{M}_O, -\mathscr{P}_W\} \\ \text{subject to} \quad & a_m \leq \mu_i \leq b_m,\ 1 \leq i \leq K \\ & a_n \leq \sigma_i \leq b_n,\ 1 \leq i \leq K \\ & 0 \leq \gamma_i,\ 1 \leq i \leq K \\ & \sum_{i=1}^{K} \gamma_i \leq E_B \end{aligned} \tag{13}$$

where $\mathbf{t} = [\mu_1, \cdots, \mu_K, \sigma_1, \cdots, \sigma_K, \gamma_1, \cdots, \gamma_K]$ is a $3K$-dimensional vector of K-primary display parameters with $\mu_1, \cdots, \mu_K$ representing peak wavelengths, $\sigma_1, \cdots, \sigma_K$ representing spectral widths for the Gaussian SPD, and $\gamma_1, \cdots, \gamma_K$ representing the relative primary optical power levels. The constraints include lower bounds, a_m and a_n, and upper bounds, b_m and b_n, for feasible search, and E_B represents the maximum power budget explored in the design space in units consistent with those for the primary power levels γ_i, $i = 1,2,\ldots K$.

Results

To obtain sample Pareto optimal designs using our proposed framework, we selected experimental settings as follows. CIE D65 was chosen[2] as the scene illuminant $l(\lambda)$, which agrees with the Rec. 2020 standard [1]. For the observers, we included the CIE 1931 2° standard observer [11], and the observers specified in a recently published Observer Function Database [17], where the CMFs of 151 color-normal observers were individually estimated. All spectral functions were represented using a 1nm sampling interval over the visible range of 400 to 700nm. Spectral distributions having a lower sampling rate were up-sampled. This relatively fine sampling allows us to represent narrow band primaries with good accuracy while maintaining a coherent framework and reasonable computational cost compared with alternative more sophisticated modeling approaches [18, 19]. For the samples of object reflectances, we selected the Macbeth DC color checker, which is commonly used for camera characterization. The flare level is set to 0.5%, i.e. $\zeta = 0.005$.

For numerical evaluation of the Pareto front, we used the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [20], which is a well-known and widely-accepted technique for applications of multi-objective optimization in several different domains. NSGA-II is an evolutionary computation based algorithm that does not require the computation of gradients, can progressively converge to the global Pareto optimal parameter set. We used the implementation of NSGA-II provided in MATLAB® via the function *gamultiobj*. We used a population size over 1000 for two-objective optimization and over 2000 for the three-objective case, and set the maximum limit on the number of generations to 5000 and used a random initialization[3]. The peak wavelengths μ_1 to μ_K were arranged in descending order and successive μ_i were constrained to be at least 5nm apart to avoid numerical instabilities and to speed up the optimization by eliminating redundant representations.

To better understand how one objective affects another, we decompose the 3-objective optimization into 2-objective cases. We allow a rather generous energy budget $E_B = 2000$ to allow exploration of the extremes of the design space, even though these may be impractical. The $\mathscr{P}_W$-$\mathscr{G}_{uv}$ (Power vs Gamut area) Pareto fronts for optimal primary designs for systems with $K = 3,4$, and 5 primaries, are shown in Fig. 2, where it can be appreciated that: (1) the larger the gamut the higher the optical power required, and (2) increasing the number of primaries widens the display gamut area, as long as an adequate level of optical power is allowed. Specifically, the figure shows that including a fifth primary pro-

[2]The SPD peak was normalized to 1, which pegs the relative optical power we report to an absolute scale.

[3]Minor refinements were seeded with the results from past optimization runs.

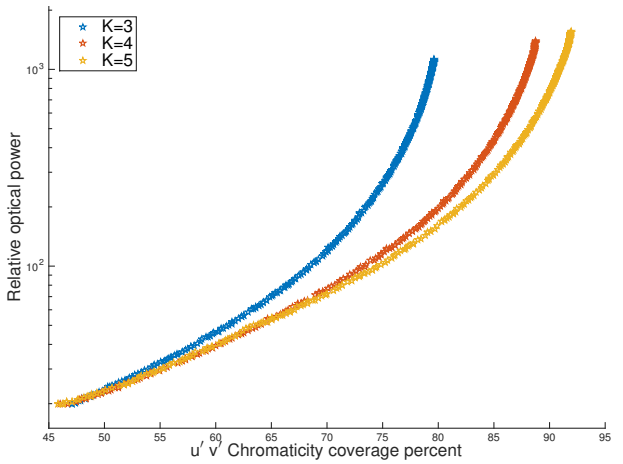

Figure 2. *Pareto fronts characterizing the tradeoff between power consumption and CIE $u'v'$ chromaticity gamut area coverage. The curve formed by the scatter-plot of points in each color defines the Pareto optimal front for these two display traits for a given number of primaries (see legend). The points at which the fronts corresponding to different numbers of primaries merge indicate thresholds for optical power (on the y-axis) below which adding primaries beyond the smallest K value among the merged curves ineffective in expanding the gamut.*

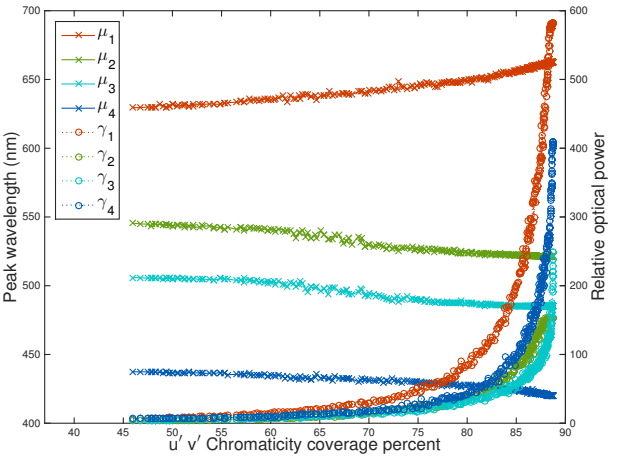

Figure 3. *Distributions of primary parameters for a four-primary display along the Pareto optimal front shown in Fig. 2 (arranged in order of increasing gamut area). For each feasible value of the $\mathscr{G}_{uv}$ along the Pareto front in Fig. 2, there are eight corresponding relevant parameters comprising the peak wavelengths and primary amplitude or relative optical power for the Gaussian spectral profiles. For one primary, the two parameters are shown in identical color and differentiated by different line styles.*

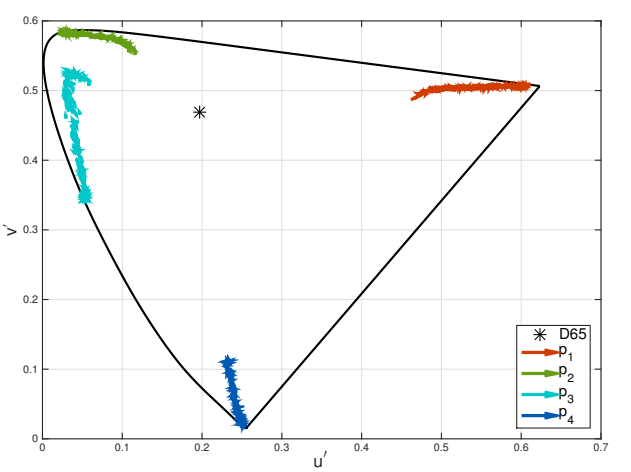

Figure 4. *Channel chromaticity trajectories for a four-primary display along the Pareto optimal front shown in Fig. 2. Each channel chromaticities correspond to the primary along with the flare. All the channels move towards to the spectral locus for enabling a larger gamut.*

vides a significant increase in gamut area coverage beyond the optimized four primary system, only if the total relative power is higher than 10^2 units. Below this threshold for power, the optimized five primary system provides the same gamut area coverage as the optimized four primary system.

An analysis of the parameters for the optimal primary designs on the Pareto front provides insights about the nature of the tradeoffs. The σ_i values are quite small for all designs on the Pareto front irrespective of the number of primaries, implying that optimal designs correspond to narrow band primaries. This also suggests that the parameters determining the values of the metrics along the Pareto fronts are the peak wavelength μ_i and the primary power γ_i. Figure 3 shows the optimal parameters for display systems with $K = 4$ primaries along its Pareto front (arranged in order of increasing gamut area coverage). It can be observed that for the power-gamut Pareto optimal designs, as the gamut area coverage $\mathscr{G}_{uv}$ increases, two of the peak wavelengths move toward the extremes of the visible region of spectra, while the other two are centered in the middle of the range of visible wavelengths. As one approaches the limiting value of the maximum gamut area coverage, the γ_i's representing the relative power of the primaries increase rapidly. This increase is particularly marked for the two primaries whose peak wavelengths λ_i have values approaching the extreme wavelengths in the visible range. Compared to the other primaries, a higher power is required to counter the impact of the flare when one wishes to push these primaries outward in chromaticity to increase the gamut area coverage. Note that the trend indicates that the "ideal" gamut area coverage would push peak wavelengths to 400nm and 700nm, but that would require a significant amount of primary power σ_i, and thus such design would not be practical, and it would not be reached under the power limit constraint E_B that we impose on our designs. For better visualization, in Fig. 4, we also show the trajectories for each of the (effective) primary chromaticities along the Pareto front for the Pareto optimal parameter values plotted in Fig. 3.

Fig. 5 presents the Pareto fronts for observer metamerism $\mathscr{M}_O$ versus gamut area $\mathscr{G}_{uv}$ showing, as expected, that the two objectives conflict with each other: reducing observer metamerism penalizes gamut area, while increasing gamut comes at the cost of deteriorating observer metamerism. The graph also shows that multiprimary displays have a clear advantage for optimizing each of the objectives and for achieving better compromises. For instance, given a design requirements for a specific level of gamut coverage, the display with more primaries performs better in the minimization of observer metamerism. Note also that the Pareto fronts can be helpful to determine the minimum number of pri-

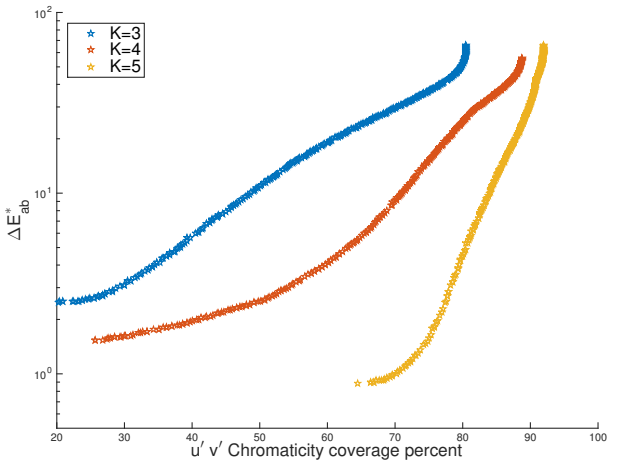

Figure 5. *Pareto fronts characterizing the tradeoff between observer metamerism and CIE $u'v'$ chromaticity gamut area coverage . Note that for design situations where the number of primaries is likely to be under 5, there is a strong tradeoff between the two traits: primaries that maximize the gamut area coverage exhibit a high degree of observer metamerism, and vice versa. With increasing number of primaries, the tradeoff between the two traits eases.*

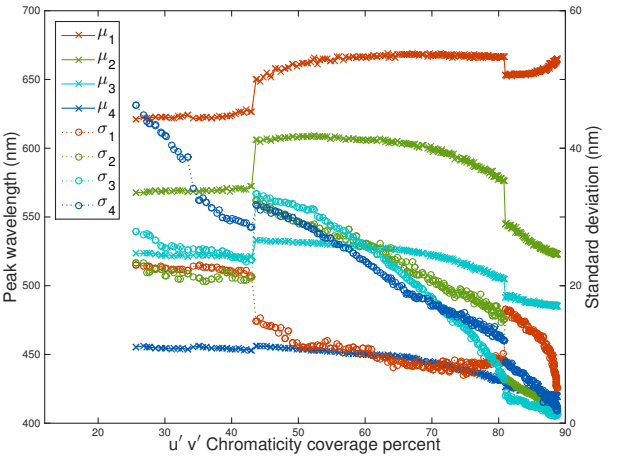

Figure 6. *Distributions of primary parameters for a four-primary display along the Pareto optimal front shown in Fig. 5 (arranged in order of increasing gamut area coverage). Peak wavelength and standard deviation for one primary are shown in identical color and differentiated by different line styles.*

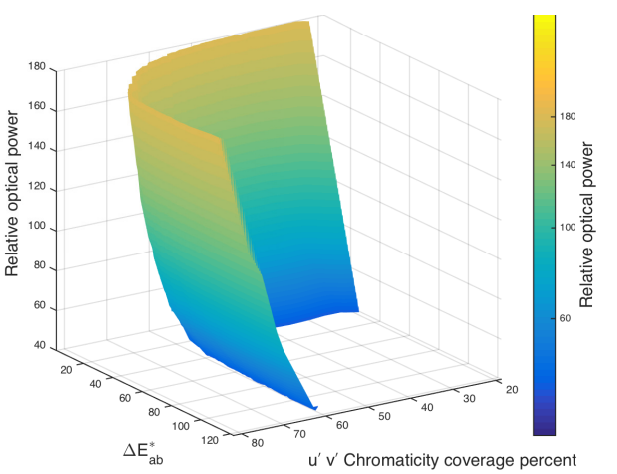

Figure 7. *The Pareto front surface for observer metamerism $\mathscr{M}_O$, gamut chromaticity coverage $\mathscr{G}_{uv}$, and optical power $\mathscr{P}_W$, for a four-primary display. Each power level can be seen as a power budget constraint, and the corresponding slice is 2D Pareto front shown earlier. As the $\mathscr{P}_W$ increase, there is Pareto improvement for $\mathscr{M}_O$ and $\mathscr{G}_{uv}$, meaning that they can be improved simultaneously with greater power consumption.*

maries to satisfy requirements for the two objectives. For example, an observer metamerism index under 1 ΔE^*_{ab} with a chromaticity gamut coverage over 70% can only be obtained by using at least $K = 5$ primaries. We also note here that these results are influenced by our choice of a display control strategy that minimizes spectral mean squared error rather than colorimetric error, which has a particularly large adverse impact on the colorimetric accuracy and the closely related observer metamerism index when the number of primaries is 3 or 4.

Fig. 6 shows the optimal spectral parameters for the $\mathscr{M}_O - \mathscr{G}_{uv}$ Pareto front for four-primary designs (arranged in increasing gamut area coverage order). The primary bandwidths σ_i exhibits greater variation along this Pareto front, suggesting that these are the main parameters driving the tradeoffs between the two objective functions: a larger gamut requires narrow band primaries, while lower observer metamerism requires wide band primaries. The peak wavelengths μ_i show some "discontinuous" jumps that appear anomalous at first glance but upon examining the effective primary trajectories in the chromaticity plane (figure not shown here) can be seen to correspond to switches between alternative geometric configurations of the primaries that provide optimal gamut area coverage as the metamerism metric is varied. The μ_i values for maximum gamut area coincide with the result in Fig. 3. Interestingly, for minimum observer metamerism, three peak wavelengths among the four primaries are approximately coincident with the prime wavelengths of 450nm, 540nm and 605nm [21], which is also a consistent finding in our 3-primary and 5-primary results. The parameters γ_i all remain stable across the Pareto front subject to the E_B constraint, since higher power supply is favorable, and thus irrelevant, for both $\mathscr{M}_O$ minimization and $\mathscr{G}_{uv}$ maximization.

A comprehensive analysis of the tradeoffs can be obtained by considering the three metrics at the same time. From the two dimensional Pareto analysis we conclude that access to power represents the main limitation for the optimization of both gamut area and observer metamerism. Therefore, we focus the exploration of the three dimensional Pareto front to the more practically meaningful range of power $40 \leq E_B \leq 180$. In this case, the Pareto front for a four primary display is represented by a three dimensional surface (the computed scatter plot is interpolated for display) shown in Fig. 7.

Conclusion and Discussion

We have presented a multi-objective optimization framework for primary design where the optimal tradeoffs between different display metrics are analyzed via the corresponding Pareto fronts. In particular, we considered three different objectives: maximizing gamut coverage, minimizing power consumption, and mini-

mizing observer metamerism. We applied our framework to simultaneously optimize pairs of objective functions, as well as the three objectives. For the 2-objective optimization, the Pareto fronts are useful for understanding the tradeoffs between the objectives, and to relate them to the spectral characteristics of the primaries, based on a simplified model of primary SPDs. The Pareto fronts for the 3-objective optimization facilitate a more comprehensive analysis, which is useful for practical decisions.

Results showed that multiprimary systems have a significant advantage for co-optimizing the conflicting objectives of gamut coverage and observer metamerism, as compared to traditional three primary displays. For a Gaussian model for the spectral power distribution of the primaries, the primary bandwidth is the major spectral parameter of the primaries that governs the compromise between both objectives. The advantages of using more primaries for co-optimizing power and gamut are only seen after reaching certain level of power, and the parameters of peak wavelength and amplitude are the main parameters that distinguish different Pareto optimal solutions.

The explorations in this paper assumed a display control strategy designed to minimize mean squared spectral error. The results demonstrate that this strategy can be effective only with an adequate number of primaries. Using our proposed multi-objective optimization framework, one can also explore the co-optimization of the display control strategy along with the primary selection. Given that such co-optimization is much more computationally demanding, particularly for the multiprimary setting, we defer this to future work.

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Author Biography

Hao Xie is a graduate student in the Electrical and Computer Engineering Department at the University of Rochester, where he is currently pursuing his Master of Science (MS) degree. He received his B.Eng. degree in information engineering from Zhejiang University, China. His research interests include color science and computational photography.