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THIN-PLATE SPLINES FOR PRINTER DATA INTERPOLATION

Gaurav Sharma and Mark Q. Shaw+*

*ECE Dept., Univ. of Rochester,
Rochester, NY, USA

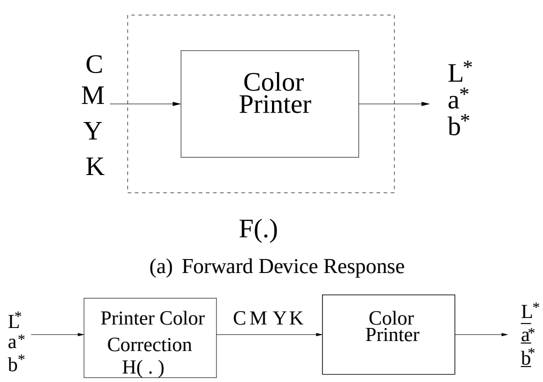
+ Hewlett Packard Company,
Boise, ID, USA

ABSTRACT

Thin-plate spline models have been used extensively for data-interpolation in several problem domains. In this paper, we present a tutorial overview of their theory and highlight their advantages and disadvantages, pointing out specific characteristics relevant in printer data interpolation applications. We evaluate the accuracy of thin-plate splines for printer data interpolation and discuss how available knowledge of printer's physical characteristics may be beneficially exploited to improve performance.

1. INTRODUCTION

The majority of present day color imaging systems use color management based on the principles of device independent color. In such an environment, device color characterization [1] is a necessary step for ensuring a stable and desired response. For a typical color printer, the characterization process consists of two main steps illustrated schematically in Fig. 1 for a CMYK printer. In the first step (Fig. 1(a)), using appropriate measurements, the (forward) device response is estimated which allows one to determine the color that the printer produces in response to a given CMYK input, where a device independent color space such as CIELAB [2] is used to represent color values numerically. Once the forward response is available, an "inverse" is determined for the purpose of correction. The correction transform is incorporated in front of the printer as shown in Fig. 1(b) and transforms color specifications in device independent coordinates into CMYK signals required by the printer to produce corresponding colors.



(a) Forward Device Response

(b) Forward Device Response

Figure 1: Printer color characterization: (a) Forward Device Response and (b) Color Correction using Inverse

For fast implementation, the forward and inverse transformations for device color correction are stored as look-up-tables. Measurement and storage restrictions mandate that only a sub-sampled version of the look-up tables can be stored and values at other nodes need to be determined using interpolation. For instance, for a CMYK printer addressed using 8 bits for each of the 4 channels, there are $\left(2^8\right)^4 = 4 \times 2^{30}$ possible, which would require for 8 bit per CIELAB coordinate, 12MB of space and in excess of 4×10^9 measurements for exhaustive measurement of the response. Similar arguments apply for the inverse transformation. Interpolation of the forward and inverse device responses is therefore a fundamental ingredient in color imaging systems. In our work, we will consider particularly the forward interpolation problem, wherein the device response for a small subset of CMYK values is measured and the response for other CMYK values must be determined through interpolation. We consider particularly the application of thin-plate splines [3] to the printer data interpolation problem.

The rest of this paper is organized as follows. In Section 2, we present related work and motivate the exploration of alternate interpolation methods. In Section 3 we review the theory of splines and illustrate using a simple 1-D example how smoothing splines can offer advantages over interpolating splines in noisy data interpolation problems such as the problem at hand. In Section 4 we review the theory of thin-plate splines and discuss the constraints and characteristics of this powerful smoothing spline interpolation method. In Section 5, we present experimental results demonstrating the application of thin-plate spline interpolation to the (forward) printer-data interpolation. Finally in Section 6, we end with some discussion and concluding remarks.

2. RELATED WORK

Various methods have been reported in the literature for the purpose of (forward) printer data interpolation. These fall in two broad classes: methods based on physical models of the print process and methods using interpolation or empirical data fitting. The modeling techniques primarily use the Neugebauer model [4, 5] and several of its empirical variants [6, 7, 8, 9, 10]. The most prominent among the latter class is the cellular Neugebauer model, which may also be viewed as a hybrid approach that combines aspects of modeling and interpolation. These methods have the advantage that they exploit the physical characteristics of the printing process and can therefore offer a fairly good representation of the device response using only a few measurements. They however have the disadvantage that in order to obtain the model parameters, access is required to the "raw" device, i.e., the capability to print using the actual control values used to drive the device (For example, the CMYK values in the case of a typical CMYK printer). This may often be unavailable and additionally some of the measurements required for the modeling, e.g. 100% each of C, M, Y, and K; may be infeasible due to physical limitations (e.g flow of inks in the case of ink-jet and peeling of toner in the case of printers). These limitations and inherent model inaccuracies sometimes limit the utility of model based methods.

Interpolation/data-fitting methods work without *a priori* knowledge of the device (apart from some assumptions of smoothness) and therefore do not encounter the same limitations. For modestly accurate representations, however, they typically require more measurements than the model-based methods. This was once a severe limitation but is not as severe a concern any more because of the availability of automated and faster color measurement instrumentation. Several interpolation approaches have been proposed for use in color conversion (see [11] for a survey and [12, 13] for additional examples). In addition, a number of data-fitting methods have been proposed that are similar to interpolation but offer the potential for some smoothing/regularization of the measured data, which is often desirable. Common methods in this class are inverse distance-weighted interpolation methods [14, 15] and locally-weighted polynomial regression [16].

One aspect that both the model-based methods and the interpolation methods handle only to a limited extent is the noise in the measurement data that impacts the interpolation nodes/model data. For the model based methods, least-squares regression [10] and total-least-squares regression [9] have been proposed as methods for partly reducing the noise in measured data used as part of the model, though fundamentally the methods rely on noiseless data assumptions. Strict interpolation based methods, by definition, ensure perfect reconstruction at the sample points and therefore also implicitly assume these are noiseless. Distance-weighted and locally-weighted polynomial regression allow for some smoothing in interpolation process but the amount of smoothing must be determined by empirical evaluation and often impacts the accuracy of the representation too due to restrictions it places on the resulting functions.

In this paper, we consider the use of thin-plate-splines [3, 17] for printer data interpolation. These are based on a framework that combines smoothing and interpolation and allow for a principled estimation of the smoothing parameter based on cross-validation [17, 18]. We first begin by a brief review of the well-known 1-D spline interpolation and illustrate using an example from printer color calibration, how this class of methods can offer an advantage in printer data interpolation.

3. SPLINES

The generic term "spline" is used to refer to a wide class of functions that are used in applications requiring data interpolation and/or smoothing[1]. Splines may be used for interpolation and/or smoothing of either one-dimensional or multidimensional data. Spline functions for interpolation are normally determined as the minimizers of suitable measures of roughness (for example integral squared curvature) subject to the interpolation constraints. Smoothing splines may be viewed as generalizations of interpolation splines where the functions are determined to minimize a weighted combination of the average squared approximation error over observed data and the roughness measure. For a number of

[1]Parts of this section borrow from contributions by the first author to wikipedia.org.

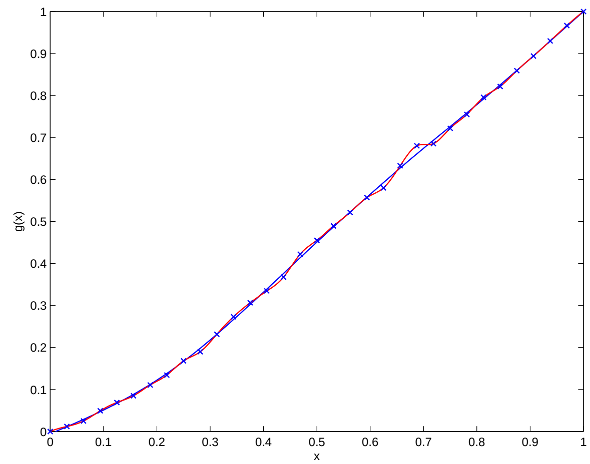


Figure 2: Interpolating Spline vs Smoothing Spline.

meaningful definitions of the roughness measure, the spline functions are found to be finite dimensional in nature, which is the primary reason for their utility in computations and representation.

In common usage, the term "spline" is often used to refer to the restricted setting of one-dimensional polynomial splines, where it refers to piece-wise polynomial functions. Often the piece-wise polynomials are cubic and subject to continuity and continuity of first derivative constraints at the knots (piecewise boundaries) resulting in the common cubic B-splines. Both interpolating and smoothing versions are feasible, a comparison of these is shown in Figure 2. Note that the example illustrates that in typical noisy data, the smoothing version is often preferable.

4. THIN PLATE SPLINES

We now review the general theory of thin plate splines. Given a set of n points $\{\mathbf{t}_i\}_{i=1}^n$ in $\mathbb{R}^d$ and a set of experimentally measured values $\{v_i\}_{i=1}^n$ at these locations, we would like to determine a function $g(\mathbf{t})$ that satisfies the dual requirements of being close to the observed data points, while at the same time being as smooth as possible. The problem may be formulated as the minimization of the penalized sum of squares

$$\mathscr{L}_{m,d}(g) = \sum_{i=1}^{n} (g(\mathbf{t}_i) - v_i)^2 + \lambda J_m(g) \qquad (1)$$

where $\lambda > 0$ and $J_m(g)$ is a smoothness penalty in d dimensions on the function $g()$ that is defined as a suitably weighted sum of integrals of squares of all the mixed derivatives of $g()$ of (total) order m. Specifically, $J_m(g)$ is mathematically defined as

$$J_m(g) = \int_{\mathbb{R}^d} \sum_{\substack{l_1+\cdots+l_d=m \\ l_i \text{ integer}, l_i \geq 0}} \frac{m!}{l_1!\cdots l_d!} \left(\frac{\partial^m g}{\partial t_1^{l_1} \cdots \partial t_d^{l_d}} \right)^2 d\mathbf{t} \qquad (2)$$

For a set of non-negative integers, $\{l_i\}_{i=1}^n$, we denote the vector $\mathbf{l} = (l_1, l_2, \ldots l_d)$ and $s(\mathbf{l}) = l_1 + \cdots + l_d$. Then we can write a corresponding differential operator

$$D^{\mathbf{l}} = \frac{\partial^{s(\mathbf{l})}}{\partial t_1^{l_1} \cdots \partial t_d^{l_d}}, \tag{3}$$

which has a order $s(\mathbf{l})$. Using this abbreviated notation the smoothness penalty can be compactly written as

$$J_m(g) \quad = \quad \int_{\mathbb{R}^d} \sum_{\mathbf{l}:s(\mathbf{l})=m} \binom{m}{\mathbf{l}} \mid D^{\mathbf{l}} g(\mathbf{t}) \mid^2 d\mathbf{t} \tag{4}$$

where $\binom{m}{\mathbf{l}}$ denotes the multi-nomial coefficient $\frac{m!}{l_1!\cdots l_d!}$. In this form, one can readily see that the the smoothness penalty is similar in form to the norm for Sobolev spaces (see for instance [19, pp. 281]) and is in fact [3] a *semi-norm* in the Sobolev space $H^m(\mathbb{R}^n)$.

Duchon [3] and Meinguet [20] demonstrated that provided $2m > d$ and the points $\{\mathbf{t}_i\}_{i=1}^n$ satisfy reasonable constraints (described later), there is a unique function that minimizes $\mathscr{L}_{m,d}(g)$ in (1) given by

$$g(\mathbf{t}) = \sum_{j=1}^{M} a_j \phi_j(\mathbf{t}) + \sum_{i=1}^{n} w_i U_{m,d}(\parallel \mathbf{t} - \mathbf{t}_i \parallel) \tag{5}$$

where $M = \binom{(m+d-1)}{d}$, $\{\phi_j\}_{j=1}^M$ are a set of polynomials spanning the M-dimensional space of polynomials in $\mathbb{R}^d$ of total degree less than m, the function $U_{m,d}()$ is given by

$$U_{m,d}(r) = \begin{cases} \theta_{m,d}\ r^{2m-d} \ln(r) & \text{if } 2m-d \text{ is an even integer} \\ \theta_{m,d}\ r^{2m-d} & \text{otherwise} \end{cases} \tag{6}$$

where

$$\theta_{m,d} = \begin{cases} \frac{(-1)^{d/2+1+m}}{2^{2m-1}\pi^{d/2}(m-1)!(m-d/2)!} & \text{if } 2m-d \text{ is an even integer} \\ \frac{\Gamma(d/2-m)}{2^{2m}\pi^{d/2}(m-1)!} & \text{otherwise} \end{cases} \tag{7}$$

The coefficients $\mathbf{a} = [a_1, a_2, \ldots a_M]$ and $\mathbf{w} = [w_1, \ldots w_n]^T$ are uniquely specified by

$$\begin{bmatrix} \mathbf{E} + \lambda \mathbf{I} & \mathbf{T}^T \\ \mathbf{T} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{w} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{0} \end{bmatrix} \tag{8}$$

where $\mathbf{v} = [v_1, \ldots v_n]$, $\mathbf{E}$ is the $n \times n$ matrix whose ij^{th} entry $E_{ij} = U_{m,d}(\parallel \mathbf{t}_i - \mathbf{t}_j \parallel)$, and $\mathbf{T}$ is the $M \times n$ matrix whose ij^{th} entry $T_{ij} = \phi_i(\mathbf{t}_j)$. In order to use TPS the above system of equations must be solved. Note that the system is a $(n+M) \times (n+M)$ system of equations, which requires roughly $(n+M)^3$ computational operations for solution and $(n+M)^2$ memory. Both these can grow quite rapidly as n increases. The requirements can be slightly reduced slightly by using the special structure of the system of equations in (8).

The smoothing parameter λ determines the trade-off between the least-squares term for fidelity to the measured data and the smoothness penalty $J(g)$. If some *a priori* knowledge is available regarding the noise in the empirically observed data, it may be used to estimate λ. Typically, however, such information is not available and one has to determine the parameter from the data itself. Generalized cross-validation methods [17, 18] are typically used to estimate an optimal value of λ efficiently.

In order for the above result to hold, the points $\mathbf{t}_i$ must be distinct and sufficiently dispersed to determine a unique[2] least-squares polynomial of total degree $m-1$.

A function of the form in (5) satisfying the constraint $\mathbf{Tw} = 0$ (note that this constraint is enforced in (8)) is called a *natural thin plate spline of order m*. Note that the first term in the expression in (5) corresponds to a global polynomial term of total degree less than m and the second term is formed by translates of the radial-basis functions $U_{m,d}()$. Thus the thin-plate spline method is closely related to radial basis function methods [21].

5. EXPERIMENTAL RESULTS

In order to quantify the performance of thin-plate spline interpolation methods in color printer data interpolation applications we conducted experiments using a four colorant laser color printer. For the experimental procedure, the data required for different approaches are obtained using a training target. The performance of the method is then evaluated over an independent test target. Spectral measurements are done using a Gretag Macbeth Spectrolino device. The device reports the data for the range 380-730 nm in 10 nm increments. For the training set, the experiments utilized CMYK input grids of sizes $17 \times 17 \times 17 \times 9$, $9 \times 9 \times 9 \times 9$, $5 \times 5 \times 5 \times 5$, and $3 \times 3 \times 3 \times 3$.

The test target consisted of an independent set of $16 \times 16 \times 16 \times 8$ CMYK patches placed within a lattice in CMYK space with random placement of each point within the corresponding lattice cell. The color values for the test set were obtained from measurements of corresponding prints and compared against the values obtained from interpolation. The difference or "error" between corresponding patches was calculated in ΔE^*_{ab} and ΔE^*_{94} units. Color error statistics for the thin plate spline interpolation technique in ΔE^*_{ab} and ΔE^*_{94} metrics are summarized in Table 1 and 2, respectively. From the numbers we see the expected trend that the errors reduce with increasing number of points. For comparison, the numbers for comparable grid sizes using a local regression technique are also included. Note that the results are comparable - however the smoothing parameter for local regression was determined manually by trial and error so as (approximately) to minimize the mean ΔE^*_{ab} error, whereas the smoothing parameters for the thin plate spline interpolation was obtained automatically. We also note that for reasonable measurement grid sizes of $5 \times 5 \times \times 5 \times 5$, the mean errors in interpolation are small enough to be reasonable for several applications.

6. DISCUSSION AND CONCLUSION

In this paper we presented an overview of thin plate splines and studied the performance of this smoothing interpolation method for interpolation of the forward device response for a laser color printer. We demonstrated that the method can offer fairly accurate results when sufficient data points are

[2]Consider the M monic monomials with total degree less than m, i.e., terms of the type $t_1^{l_1} \cdots t_d^{l_d}$, where each l_i is a non-negative integer and the total order $l_1 + \cdots + l_d < m$. At each point $\mathbf{t}_i$, an $M \times 1$ vector may be formed by evaluating each of these terms in a specifi c order. If the $M \times n$ matrix formed by the concatenation of each of these vectors corresponding to the n points has a rank equal to M, then for any set of values $\{v_i\}_{i=1}^n$ there is a unique polynomial $p(x)$ with total degree $\leq (m-1)$ that minimizes $\sum_{i=1}^n (p(x) - v_i)^2$. Note that a direct implication is that the method requires $n > M$ points.

Method	Data Grid	Mean	Standard Deviation	95 percentile	Maximum
Thin Plate Spline	9×9×9×9	1.4893	1.9064	3.3378	25.5749
	5×5×5×5	1.8042	1.9520	4.2092	25.4651
	3×3×3×3	4.4604	2.8103	9.7720	24.7110
Local Regression	$9 \times 9 \times 9 \times 9$	1.6188	1.9249	3.7884	24.9900
	$5 \times 5 \times 5 \times 5$	1.9945	2.1007	4.7518	25.3187
	$3 \times 3 \times 3 \times 3$	3.5649	2.4666	8.1184	27.3989

Table 1: Color error statistics (in ΔE^*_{ab} units)for different empirical interpolation methods as a function of grid size

Method	Data Grid	Mean	Standard Deviation	95 percentile	Maximum
Thin Plate Spline	9×9×9×9	1.1076	1.1909	2.5557	15.9328
	5×5×5×5	1.3151	1.2048	2.9980	15.9055
	3×3×3×3	3.1351	1.8669	6.8912	21.8168
Local Regression	$9 \times 9 \times 9 \times 9$	1.1827	1.1963	2.7190	15.7596
	$5 \times 5 \times 5 \times 5$	1.4452	1.2911	3.2806	15.5316
	$3 \times 3 \times 3 \times 3$	2.5128	1.5648	5.4841	17.5934

Table 2: Color error statistics (in ΔE_{94} units)for different empirical interpolation methods as a function of grid size

utilized. One dimensional examples illustrated the benefit of the technique in smoothing noise in the measured data.

An open problem of some interest in splines is the optimal placement of spline knots. In our present work, this aspect was not addressed. Suitable placement of the knots can, however, significantly improve the accuracy of the interpolation methods and is therefore worthy of independent investigation. In future work, we will explore how the knowledge of the printer characteristics may be exploited for this purpose.

7. ACKNOWLEDGMENT

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