Stochastic Screens Robust to Mis-Registration in Multi-Pass Printing

Gaurav Sharma\textsuperscript{a*}, Zhigang Fan\textsuperscript{b}, Shen-ge Wang\textsuperscript{b}

\textsuperscript{a}ECE Dept., Univ of Rochester, Rochester, NY 14627-0126
\textsuperscript{b}Xerox Corporation, Webster, NY/USA

Abstract

A new technique for design of stochastic screens is proposed that produces screens that are robust against mis-registration in multi-pass printing. Conventional stochastic screens are designed through an optimization process that minimizes low-frequency structure in halftone images under the assumption that the placement of pixels is accurate. In inkjet printing, however, a page is often printed in multiple passes to allow for better drying of inks and to minimize appearance of a head signature. Any potential mis-registration between the passes is typically not comprehended in the conventional stochastic screen design process. The mis-registration between the passes can therefore cause significantly increased graininess (low-frequency structure) in printed images produced with stochastic screens even though the corresponding electronic bitmaps are free from low-frequency structure. In this paper, we propose modifications to the stochastic screen design process that take the two pass printing into account and produce halftones that are robust to inter-pass mis-registration errors. This allows reduced tolerances and alignment requirements in manufacturing that translate to lower cost. The proposed technique works by modifying the screen design process to ensure that a majority of the minority pixels are concentrated in a single pass, which provides improved robustness to mis-registration between the passes. Experimental results demonstrate that the proposed design technique performs significantly better than conventional stochastic screens in the presence of mis-registration errors.

Keywords: stochastic screen, mis-registration, halftoning, halftone design

1. Introduction

Stochastic halftone screens are commonly used for digital halftoning \cite{1-4}. While their performance is typically poorer than adaptive halftoning techniques such as error-diffusion and model based halftoning, they offer very significant computational advantages making them preferable in applications where processing time or cost is a consideration. In particular, they are commonly used in inkjet printing devices widely used for both black and white and color printing.

The design of stochastic screens attempts to minimize the low frequency texture in halftone images that are produced from the screening process. Well designed stochastic screens produce dispersed dot (FM) halftones, that have minimal low-frequency structure. While this is true of the digital bit maps produced by stochastic screens, the appearance of printed images is also dependent on other components of the printing system. In particular, inkjet printers often use a print mode with two passes in order to reduce the visibility of head signature and to allow better drying of inks. The pixels on the page are spatially partitioned into two sets with one set being printed in each pass (often, one pass is printed in the forward direction of head traversal and the other in the

reverse direction). For the rest of this paper, the partition is assumed to be a “checker-board” partition, the ideas developed are, however, equally applicable to other partitions that may be chosen (for example, an alternate line partition or a stochastic partition). The checkerboard partition, which is commonly used in practice, is shown graphically in Figure 1, where each of the black/white squares correspond to a pixel. The white pixels correspond to one partition, which we will refer to as Partition 1, and the black ones to the other partition, referred to as Partition 2 in this paper.

![Figure 1. Checkerboard Partition](image)

In printing an image on the page, the printer prints the pixels corresponding to (say) partition 1 in the first pass and to partition 2 in the second pass. If the registration between the passes is perfect, the graininess of the resulting images is largely unchanged in comparison with a printer that prints the entire image in a single pass. However, if there is mis-registration between the two passes, undesired textures may be generated that result in considerably increased graininess in the print. Examples of this problem are demonstrated in Figures 4 and 5. Figure 4 is a halftone image obtained using a stochastic screen where there is no mis-registration between the checkerboard partitions and Figure 5 is a simulation of the $(\Delta x = 1 \text{ pixel}, \Delta y = 1 \text{ pixel})$ mis-registration. Note the increased graininess and undesirable textures in Figure 5 in comparison to Figure 4.

For the above images, the mis-registration was simulated electronically. In actual inkjet printers, the misregistration arises from mechanical positioning errors between the two passes. While increased precision in mechanical positioning would mitigate the problem, it could involve significant cost due to the tight tolerances required particularly at high resolutions. If the mis-registration is identical from page to page and over the life of the printer, it can be detected a priori and can be compensated for electronically. However, electronic compensation of each individual printer adds cost to these low-end devices and cannot correct registration errors under half a pixel (without excessive computation).

We propose modifications to the stochastic screen design process to make it more robust to mis-registration between the two passes. The stochastic screens designed using the modified process provide increased robustness to inter-pass registration errors by primarily using the pixels from a single partition for printing in the highlight regions where graininess is the biggest problem. By concentrating the minority black pixels in a single pass, these methods ensure that the gap between these minority pixels is not affected by inter-pass mis-registration errors. For the proposed methods, a similar benefit is also obtained in the shadow regions by similarly ensuring that the minority white pixels are located in a single pass. Modifications to error diffusion with similar motivation have previously been proposed in [5].

This paper is organized as follows. In Section 2, an overview of the stochastic screen design process is presented. Section 4 describes the modified design process. Section 4 shows simulation results and a summary is given in Section 5.

**2. Stochastic Screen Design**

A Stochastic screen is an MxN array of thresholds values that is used for halftoning. The stochastic screen is overlaid on the image to be halftoned. The screen is replicated along either spatial dimension to cover the spatial extent of the image (which is typically much larger than the screen). At each pixel, the output is set to a black or
white value based on a comparison of the screen to the image. For our purposes, we assume that an output pixel is set as black if the image value exceeds that of the screen at the same location and white otherwise.

For our discussion here, we focus on halftoning of single channel images— the technique can however be extended to color. Let \( i(m,n), s(m,n), \) and \( b(m,n) \) denote the input image, the stochastic screen, and the output bitmap at pixel \((m,n)\). It is assumed that \( b(m,n) \) has a binary value of 0 (white) or 1 (black), and \( s(m,n) \) and \( i(m,n) \) take on values in the range of \([0,1]\). The halftoning screening process defines the output in terms of the screen and the input image as:

\[
b(m,n) = \begin{cases} 
1 & \text{if } i(m,n) > s(m,n) \\
0 & \text{if } i(m,n) \leq s(m,n) 
\end{cases}
\]

From the definition, it is clear that \( s(m,n) \) may be thought of as the value at which the pixel at location \((m,n)\) turns to an on (black) state. Since the screen is replicated to tile the image plane, only one \( M \times N \) period of the screen will be considered in the following discussion. The replication is however accounted for in the design process.

Consider the halftoning of a series of images each having a size \( M \times N \) and having a constant value over its spatial extent. For an image value of 0, all pixels are white (0). As the image value increases it exceeds the screen thresholds at a number of locations creating a corresponding number of black pixels. For our discussion here we further assume that all screen values are distinct so that single pixels are successively turned on. Under this assumption, the halftone screen \( s(m,n) \) can equivalently be represented by the unique sequence of pixel locations \( \Psi = \{(x(k),y(k))\}, k = 1, 2, \ldots MN \) corresponding to the order in which the pixels of the \( M \times N \) halftone screen are turned on as the input increases from 0 to 1. Thus as the input increases from 0 towards 1, there are \( MN \) distinct patterns produced, the \( r^{th} \) pattern in the sequence has the pixel locations \((x(1),y(1)), (x(2),y(2)), \ldots, (x(r-1), y(r-1))\) as black and the pixel locations \((x(r),y(r)), (x(r+1),y(r+1)), \ldots, (x(MN),y(MN))\) as white.

A desirable property of the halftone screen (in the absence of mis-registration considerations) is that the minority pixels at any given gray level are evenly distributed. The property can be mathematically described in terms of the minimization of an objective function that measures the low-frequency content in halftone output textures. For our discussion, we consider a specific form of the objective function defined in terms of the \( MN \) distinct halftone patterns that are produced.

For \( r < (MN)/2 \), the pattern has a minority of black pixels and for \( r > (MN)/2 \) the pattern has a minority of white pixels. For \( r < (MN)/2 \), the minority pixels are black and we would therefore like to maximize the distances among the black pixels. The quality contribution of the pair of pixels \((x(l),y(l)), (x(k),y(k))\) where \( l,k \) are distinct numbers between 0 and \((r-1)\), is defined as [4]

\[
Q_{lk} = \exp(-C \left(d_{lk}^2/d_{o}^r\right))
\]

where \( d_{lk} \) is the distance between the corresponding pair of points (under the assumption of replication), and \( d_{o}^r = (MN)/\max(l,k) \), and \( C \) is a constant. Likewise for \( r > (MN)/2 \), the minority pixels are white and the contribution of a pair of pixels \((x(l), y(l)), (x(k), y(k))\) where \( l, k \) are distinct numbers between \( r \) and \( MN \), can be written in the same form as (2), with \( d_{o}^w = (MN)/(MN-\min(l,k)) \), and other terms as defined earlier.

The overall merit function for the screen \( Q(\Psi) \) may be computed as the summation of the quality over all minority pairs in the black minority regime and in the white minority regime. The effect of swapping a pair of pixels in the sequence can therefore be evaluated readily in terms of the change in this merit function. The screen may therefore be designed using an optimization function to maximize \( Q(\Psi) \). Specifically for this paper we consider a simulated annealing scheme in order to accomplish this. Details for the process may be found in [4].

### 3. Incorporating Mis-registration Robustness

We propose a modification of the stochastic screen design algorithm in order to make it more robust to mis-registration between the two passes. The basic principle for maximized robustness is the idea of “minority
concentration"[5]. Specifically, the modified design ensures that all the black pixels in highlights are printed in one partition, say Partition 1, and all the white pixels in shadows are assigned to another partition (Partition 2).

The features and advantages of the “minority concentration” scheme are:

1. For uniform (or slowly varying) regions where input ≤ 0.5, all the black pixels are located in Partition 1. Entire Partition 2 is white. As a result, there are no mis-registration artifacts as shown in Figure 2.

2. When the input reaches 0.5, the entire Partition 1 is black, and the entire Partition 2 is white. The combined output is a checkerboard.

3. When input exceeds 0.5, Partition 1 remains black. Partition starts to have black pixels. The combined image is a checkerboard with some of the white holes filled (Figure 3). Mis-registration does not produce significant texture changes. However, it may introduce a reduced density, since mis-alignment may cause black pixels in Partition 1 and Partition 2 overlapping. Nevertheless, considering the dot gain, the actual density reduction may not be very severe.

The objective of minority concentration can be achieved by several different methods. We discuss a simple technique for incorporating this in screen design. An additional screen merit function is defined by considering the quality measure applied only to the individual partitions. Let $Q_1(\Psi)$ denote the merit function applied only to the pixels corresponding to partition 1 and $Q_2(\Psi)$ denote the merit function applied only to the pixels corresponding to partition 2.

![Figure 2. Output of proposed method for Input < 0.5. a) Partition 1; b) Partition 2; c) Combined output with no mis-registration; d) Combined output with mis-registration of (Δx = 0.5, Δy = 0.5)](image)

![Figure 3. Output of proposed method for Input > 0.5. a) Partition 1; b) Partition 2; c) Combined output with no mis-registration; d) Combined output with mis-registration of (Δx = 1, Δy = 1)](image)

Define a combined merit function as

$$M(\Psi) = w_1 Q_1(\Psi) + w_2 Q_2(\Psi) + Q(\Psi) \quad (3)$$
where $w_1$, and $w_2$ are suitable weights. For our purposes, we choose $w_1 = w_2 = 3$. If the design optimizes the merit function $M(\Psi_s)$, we are assured that the halftones restricted to either partition have minimal undesirable texture, and the combination of these in the overall screen also avoids undesirable textures (though to a much lesser extent).

Once a screen $\Psi_s$ is obtained from the optimization of the merit function in (3), a new screen can obtained which uses the minority concentration idea in order to achieve robustness against mis-registration. The new screen is defined by the sequence

$$\Psi^{p}_s = \Psi_s |_1 |_2 \Psi_s |_2$$

Where $|_1$ denotes the operation of restricting the sequence to the first partition while preserving the order (within the partition), and $|_2$ is similarly defined, and $|_1$ denotes the concatenation of the sequences. From the definition, it is apparent that for the screen defined by the sequence $\Psi^{p}_s$, the minority pixels in the highlights are constrained to the first partition and the minority pixels in the shadows are constrained to the second partition. Hence the screen follows the minority concentration principle outlined earlier and is therefore robust against inter-pass mis-registration.

4. Simulation Results

The results of using the above mentioned modified stochastic screen designs are demonstrated and compared against conventional designs in Figures 4-7. Figure 6 shows the halftone image obtained with the modified error diffusion method described above for the case of no inter-pass mis-registration and Figure 7 shows the corresponding image for a simulated mis-registration of $(\Delta x = 1, \Delta y = 1)$. Note that increase in image graininess as from Figure 7 to Figure 8 (if any) is much smaller than the drastic increase in graininess seen from Figure 4 to Figure 5 for conventional stochastic screens. This clearly demonstrates that the modified stochastic screen design method presented in this paper is significantly more robust to inter-pass mis-registration than conventional design process. Similar results can be seen for other mis-registration values.

References


Figure 4. Conventional method with no mis-registration.
Figure 5. Conventional method with mis-registration of $(\Delta x = 1, \Delta y = 1)$. 
Figure 6. Proposed method with no mis-registration.
Figure 7. Proposed method with mis-registration of $(\Delta x = 1, \Delta y = 1)$. 