Mathematical Discontinuities in CIEDE2000 Color Difference Computations

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Outline

• Color Difference Equations
• CI EDE2000 Computation
• Sources of Discontinuity
• Discontinuity Visualization
• Discontinuity Magnitude Characterization
  ▪ Maximum (reasonable) magnitude
• Conclusions + workarounds
Color Difference Equations

- Quantitative evaluation of color differences
- Main uses:
  - Quantitative color error evaluation
  - Algorithm/parameter optimization
Color Difference Equations: Desirable Attributes

- Perceptual uniformity
  - Equal numerical differences correspond to equal perceived differences

- Mathematical properties:
  - Continuity and differentiability
    - Taylor series/small-error approximation
    - Gradient based optimization
  - Symmetry
    - reference/sample distinction un-necessary
  - Correspondence to a distance metric
    - Underlying “uniform” color space
**CIE 1976 CI ELAB Color Space**

- “Uniform” color space
  - Based on ANLAB, in turn on Munsell
- Transformation of 1931 CI EXYZ tristimulus coordinates
- Nonlinearity: Cube-root with linear end segment
  \[
  f(x) = \begin{cases} 
  x^{\frac{1}{3}} & x > .008856 \\
  7.787x + \frac{16}{116} & x \leq .008856 
  \end{cases}
  \]
- Transformation carefully designed
  - Continuous first derivatives [Pauli1976]
Cl ELAB Based Color Difference

Fomulae

• 1976: $\Delta E_{ab}^*$ Color difference
  ▪ Euclidean distance betw. points in Cl ELAB space
  \[
  \Delta E_{ab}^* = \sqrt{\Delta L^2 + \Delta a^2 + \Delta b^2} = \sqrt{\Delta L^*^2 + \Delta C^*^2 + \Delta H^*^2}
  \]

• CMC and Cl E ‘94 color difference Eqns.
  ▪ Chroma/Hue dependent weights for $\Delta L^*$, $\Delta C^*$, $\Delta H^*$
  ▪ Greater uniformity w.r.t. experimental data
  ▪ Retain continuity of first derivatives
CI EDE2000

- **a* Axis Scaling**
  - \( a^* \rightarrow a' \)
- **Decomposition**
- **Hue, Chroma Dependent Weighting**
- **Cross Term (blue hue nonlinearity)**

\[
\Delta E_{00}^{12} = \sqrt{\left( \frac{\Delta L'}{k_L S_L} \right)^2 + \left( \frac{\Delta C'}{k_C S_C} \right)^2 + \left( \frac{\Delta H'}{k_H S_H} \right)^2 + R_T \left( \frac{\Delta C'}{k_C S_C} \right) \left( \frac{\Delta H'}{k_H S_H} \right)}
\]

- CI EDE2000 Color Difference is discontinuous
Cl EDE2000 Hue & Hue Weighting Functions

\[ \Delta H' = 2 \sqrt{C'_1 C'_2} \sin \left( \frac{\Delta h'}{2} \right) \]

\[ T = 1 - 0.17 \cos(\bar{h}' - 30^\circ) + 0.24 \cos(2\bar{h}') + 0.32 \cos(3\bar{h}' + 6^\circ) - 0.20 \cos(4\bar{h}' - 63^\circ) \]

\[ S_H = 1 + 0.015 \bar{C}' T \]

- \( C'_1, C'_2 \) sample chroma values
- \( \Delta h' \) hue angle difference
- \( \bar{h}' \) mean hue angle
- \( \bar{C}' \) mean chroma value (arithmetic)
Mean Hue/Hue Difference Computation

- Mean: Bi-sector of smaller angle between $h_1$ and $h_2$
- Difference: Smaller angle + direction gives sign

\[
\Delta h' = h_1 - h_2
\]

Discontinuous Operations
Mean Hue Discontinuity

- $180^\circ$ discontinuity in mean hue
Hue-difference Discontinuity

- $180^\circ$ (Sign) discontinuity in hue difference

$\Delta h'_{12} = \pi - \varepsilon/2$

$\Delta h'_{13} = -\pi + \varepsilon/2$

- $180^\circ$ (Sign) discontinuity in hue difference
Discontinuity Characterization

- Where does it occur?

- How big is it (magnitude)?
Discontinuity Locations

- 6-D Space of input values

\[ \Delta E_{00}(L_1^*, a_1^*, b_1^*; L_2^*, a_2^*, b_2^*) \]

- Discontinuity for points \(180^\circ\) apart in hue

\[ a_1 b_2 = -a_2 b_1 \]

- 5-D manifold in 6-D space
Discontinuity Locations

- Discontinuity loci in $h_1, h_2$ plane

\[ h_2 = h_1 + 180 \]

\[ h_2 = h_1 - 180 \]
Visualization
Discontinuity Magnitude

- Main contribution mean hue discontin. in

\[ \left( \frac{\Delta H'}{k_H S'_H} \right)^2 \]

- Minor contribution from hue diff. discontin.
  - Sign change of \( \Delta H' \)
  - Contributes through rotation term
Discontinuity Magnitude Bounds

- CIEDE2000 intended for small color differences
- Colors under 5 $\Delta E_{ab}^*$ units apart
  - Discontinuity magnitude under 0.2374
    - Non-negligible, not too large
  - Occurs for 143° hue sample
- Increasing distance: sharp rise
Conclusions

- CI EDE2000 color difference is a discontinuous function
- Discontinuity for colors 180° apart in hue
- Discontinuity magnitude small in small error practical applications
  - Under 0.238 for color under 5 $\Delta E^*_{ab}$ units apart
- Serious limitation for
  - Taylor series/small error approximations
  - Gradient based optimization
Potential workarounds/fixes

- Use formula asymmetrically
  - Major discontinuity due to mean hue eliminated
- Symmetrize if nesc by averaging color differences
- Discontin in Rotation term remains
  - Harder to fix
    - Probably requires different functional format and re-optimization of parameters
Additional Information

• Upcoming paper in Color Research and Application (Feb 2005)
  - includes detailed algorithmic statement of CIEDE2000 computation
  - Additional test data
    - Several available implementations
      + Agreement over CIE draft test data, disagreement over other data!!
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Questions