ECE III EXAM #2 REVIEW

OP-AMPS

- Basic Inverting Amplifier:
  \( v_o = A v_i = A (v_+ - v_-) \)
  \( A \approx 10^5 \)

- Voltage Follower:
  \( v_o = \frac{A}{1 + A} v_i \approx v_i \)

ENERGY STORAGE ELEMENTS

CAPACITORS:

- \( q = CV \)
- \( i = \frac{dq}{dt} = C \frac{dv}{dt} \)

- \( \int_{-\infty}^{t} i(t')dt' = v(t) - v(0) - \frac{1}{C} \int_{0}^{t} v dt' \)

- \( W_{cap} = \frac{C}{2} v^2 \) (stored energy)

SERIES:
  \( C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \)

PARALLEL:
  \( C_{eq} = C_1 + C_2 \)
**CAPACITIVE VOLTAGE DIVIDER**

\[ V_{\text{out}} = \frac{C_1}{C_1 + C_2} V_{\text{in}} \]

**CAPACITIVE CURRENT DIVIDER**

\[ i = \frac{C_1}{C_1 + C_2} i' \]

**INDUCTORS:**

\[ \lambda = L i \quad \text{so} \quad V = \frac{dx}{dt} = L \frac{di}{dt} \]

OR

\[ i = \frac{1}{L} \int_{-\infty}^{t} V(t') dt' = i(t) + \frac{1}{L} \int_{0}^{t} V dt' \]

\[ W_{\text{ind}} = \frac{1}{2} L i^2 \leftarrow \text{STORED ENERGY} \]

**SERIES**

\[ L_{\text{eq}} = L_1 + L_2 \]

**PARALLEL**

\[ L_{\text{eq}} = \frac{L_1 L_2}{L_1 + L_2} \]

\[ V_{\text{out}} = \frac{L_2}{L_1 + L_2} V_{\text{in}} \]

\[ i = \frac{L_1}{L_1 + L_2} i \]
**SUMMARY OF R's, L's, AND C's**

\[ \mathbf{R} \quad \mathbf{C} \quad \mathbf{L} \]

\[ v = Ri \quad v(t) = v(0) + \frac{1}{C} \int_0^t i \, dt \quad \mathbf{v} = L \frac{di}{dt} \]

\[ i = \frac{v}{R} = GV \quad i = C \frac{dv}{dt} \quad i(t) = i(0) + \frac{1}{L} \int_0^t v \, dt \]

**SERIES**

\[ R_{eq} = \Sigma R \quad C_{eq} = \frac{1}{\Sigma \frac{1}{C}} \quad L_{eq} = \Sigma L \]

**PARALLEL**

\[ R_{eq} = \frac{1}{\Sigma \frac{1}{R}} \quad C_{eq} = \Sigma C \quad L_{eq} = \frac{1}{\Sigma \frac{1}{L}} \]

**POWER & ENERGY**

\[ P_R = Ri^2 = \frac{v^2}{R} \quad P_C = \frac{d}{dt} \left[ \frac{1}{2} C v^2 \right] \quad P_L = \frac{d}{dt} \left[ \frac{1}{2} L i^2 \right] \]

**IRREVERSIBLE CONVERSION TO HEAT**

**STORING ENERGY**

**STORING ENERGY**

**VOLTAGE CANNOT CHANGE INSTANTANEOUSLY**

**CURRENT CANNOT CHANGE INSTANTANEOUSLY**

**TRANSIENT BEHAVIOR:**

- **Short time:** SHORT
- **Long time:** OPEN
FIRST-ORDER CKTS:
\[
\frac{dx}{dt} + \frac{x}{\tau} = f(t)
\]
x(t) = x_{ss} + x_t(t), \ t \geq 0
\[
x_t = C e^{-t/\tau}
\]

ALWAYS REDUCIBLE TO ONE EQUIV. CAPACITOR OR INDUCTOR PLUS A RESISTOR.

CAPACITIVE TRANSIENTS
\[
\begin{align*}
V_c(t) &= V_0 \left[1 - e^{-t/\tau}\right], \ t \geq 0 \\
\end{align*}
\]
\[
\gamma = R_{eq} C_{eq} = RC \\
V_c(0-) = V_c(0+) = 0 \\
V_c(t \to \infty) = V_0
\]

INDUCTIVE TRANSIENTS
\[
\begin{align*}
V_L(t) &= V_0 e^{-t/\tau}, \ t \geq 0 \\
\end{align*}
\]
\[
\tau = \frac{L_{eq}}{R_{eq}} = \frac{L}{R} \\
i_L(0-) = i_L(0+) = 0 \\
i_L(t \to \infty) = V_0
\]
LECTURE #20  10-26-07

TRANSIENT ANALYSIS

1. FIND INITIAL CONDITION (via DC model)
2. FIND FINAL CONDITION
3. FIND Ckt TIME CONSTANT
   - TURN OFF ALL SOURCES
   - REDUCE RC or LR NETWORK
   - \( \tau = \frac{R_{eq}C_{eq}}{R_{eq} + \frac{1}{C_{eq}}} \)
4. \( v_c(t) = v_c(\infty) + [v_c(0+) - v_c(\infty)] e^{-t/\tau}, \quad t \geq 0 \)
   OR
   \( i_L(t) = i_L(\infty) + [i_L(0+) - i_L(\infty)] e^{-t/\tau}, \quad t \geq 0 \)

EXAMPLES

1. \( v(t<0) = RI_0 \) so \( v(0+) = RI_0 \)
2. \( v(t \rightarrow \infty) = 0 \)
3. \( \tau = RC \)
4. \( v(t) = RI_0 e^{-t/\tau} \)

#2

1. \( v_c(t<0) = V_0 \) so \( v(t<0) = V_0 \)
2. \( v_c(t \rightarrow \infty) = 0 \)
3. \( \tau = \frac{(R_1 + R_2)C}{R_2} \)
   \( v_c(t) = V_0 e^{-t/\tau}, \quad t \geq 0 \)
4. \( v(t) = \frac{R_2}{R_1 + R_2} v_c(t) \)

\[ v(t) = \frac{R_2}{R_1 + R_2} v_0 e^{-t/\tau}, \quad t \geq 0 \]
**STEP \( \frac{1}{6} \) RAMP FUNCTIONS**

\[
u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}, \quad r(t) = t u(t)
\]

**SECOND-ORDER CKTS: RLC's**

\[
Ri + L \frac{di}{dt} + \frac{1}{C} \int i \, dt = V_s
\]

OR

\[
\frac{d^2i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = \frac{dV_s}{dt}
\]

\[
\alpha = \frac{R}{2L}
\]

**INITIAL CONDITIONS:**

- 2 ARE NEEDED
- \( i_c(0-) = i_c(0+) \)
- \( V_c(0-) = V_c(0+) \)

**TO SOLVE FOR** \( A_1, \frac{1}{2} A_2 \)}
PARALLEL RLC "TANK CKT"

\[ \frac{v}{R} + \frac{C}{L} \frac{dv}{dt} + \frac{1}{L} \int v(t') \, dt' = i \]

OR

\[ \frac{d^2 v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v = \frac{i}{C} \]

\[ x = \frac{1}{2RC}, \quad \omega_d^2 = \sqrt{\omega_0^2 - x^2}, \quad \omega_0 = \frac{1}{\sqrt{LC}} \]

SOLUTION FORM

\[ v(t) = k_1 e^{-\alpha t} \cos \omega_d t + k_2 e^{-\alpha t} \]

AGAIN, TWO INITIAL ARE NEEDED, WHICH CAN BE OBTAINED FROM \( v_c(0) = v_c(0^+) \)

\[ i_c(0) = i_c(0^+) \]

USEFUL TRICK:

FOR SHORT TIMES, CAPACITANCE BEHAVES LIKE A SHORT CKT \& INDUCTANCE LIKE AN OPEN CKT

FOR LONG TIMES, CAPACITANCE BEHAVES LIKE AN OPEN CKT \& INDUCTANCE LIKE A SHORT CKT.