Problem 9.9  Circuit (b) in Fig. P9.9 is a scaled version of circuit (a). The scaling process may have involved magnitude or frequency scaling, or both simultaneously. If $R_1 = 1 \, \text{k}\Omega$ gets scaled to $R'_1 = 10 \, \text{k}\Omega$, supply the impedance values of the other elements in the scaled circuit.

![Circuits for Problem 9.9](image)

**Solution:**

$$K_m = \frac{R'_1}{R_1} = 10.$$  

From the original circuit,

$$Z_{L_1} = j\omega L_1 = j10 \, \Omega.$$  

For the scaled circuit,

$$Z'_{L_1} = j\omega' L'_1 = j\omega L_1 \cdot \frac{K_m}{K_f} L_1 = jK_m \omega L_1 = j10 \times 10 = j100 \, \Omega,$$

$$Z'_{L_2} = j200 \, \Omega,$$

$$Z'_{C_1} = \frac{1}{j\omega'C'_1} = -jK_m \omega C_1 = -\frac{j10}{\omega C_1} = -j50 \, \Omega,$$

$$Z'_{C_2} = -j500 \, \Omega,$$

$$R'_2 = K_m R_2 = 10 \times 2 \, \text{k}\Omega = 20 \, \text{k}\Omega.$$  

![Scaled circuit](image)
Problem 9.12  Convert the following dB values to voltage ratios:

(a) 46 dB  
(b) 0.4 dB  
(c) −12 dB  
(d) −66 dB  

Solution:  
(a) $10^{(46/20)} = 10^{2.3} = 199.5 \simeq 200.$  
(b) $10^{(0.4/20)} = 10^{0.02} = 1.047.$  
(c) $10^{(-12/20)} = 10^{-0.6} = 0.25.$  
(d) $10^{(-66/20)} = 10^{-3.3} = 5 \times 10^{-4}.$
Problem 9.13  Generate Bode magnitude and phase plots for the following voltage transfer functions:

(a) \( H(\omega) = \frac{j100\omega}{10 + j\omega} \)

(b) \( H(\omega) = \frac{0.4(50 + j\omega)^2}{(j\omega)^2} \)

(c) \( H(\omega) = \frac{40 + j80\omega}{10 + j50\omega} \)

(d) \( H(\omega) = \frac{(20 + j5\omega)(20 + j\omega)}{j\omega} \)

(e) \( H(\omega) = \frac{30(10 + j\omega)}{(200 + j2\omega)(1000 + j2\omega)} \)

(f) \( H(\omega) = \frac{j100\omega}{(100 + j5\omega)(100 + j\omega)^2} \)

(g) \( H(\omega) = \frac{200 + j2\omega}{(50 + j5\omega)(1000 + j\omega)} \)

Solution:

(a) \( H(\omega) = \frac{j100\omega}{10 + j\omega} = \frac{j100\omega}{10(1 + j\omega/10)} = \frac{j10\omega}{1 + j\omega/10} \).

- Constant factor 10 \( \Rightarrow \) +20 dB
- Zero @ origin
- Simple pole with \( \omega_c = 10 \) rad/s

\[
M \,[dB] = 20 \log |H| \\
= 20 \log 10 + 20 \log \omega - 20 \log |1 + j\omega/10|
\]
\( M[\text{dB}] = 20 \log |H(\omega)| = 20 \log 1000 + 40 \log |1 + j\omega/50| - 40 \log \omega. \)

- Line starts at 60 dB at \( \omega = 1 \text{ rad/s}, \) and has slope of \(-40 \text{ dB/decade}\)
  - Constant factor 1000 \( \implies \) 60 dB
  - Pole @ origin of order 2
- Simple zero with \( \omega_c = 50 \text{ rad/s}, \) of order 2
\( \omega \) (rad/s)  
\( \phi \)  
\( -180^\circ \)  
\( 0 \)  
\( 180^\circ \)  

**Magnitude**  
-40 log \( \omega \)  
20 log 1000 = 60 dB  
40 log \( |1 + j\omega/50| \)  

**Phase**  
\( \phi(\omega) \)  
0  
-180°

\[ H(\omega) = \frac{40 + j80\omega}{10 + j50\omega} = \frac{40(1 + j2\omega)}{10(1 + j5\omega)} = \frac{4(1 + j\omega/0.5)}{(1 + j\omega/0.2)}. \]

- Constant factor 4 \( \implies \) 12 dB
- Simple pole with \( \omega_c = 0.2 \text{ rad/s} \)
- Simple zero with \( \omega_c = 0.5 \text{ rad/s} \)

\[ M \text{ [dB]} = 20 \log |H(\omega)| = 20 \log 4 + 20 \log |1 + j\omega/0.5| - 20 \log |1 + j\omega/0.2| \]
\[ H(\omega) = \frac{(20 + j5\omega)(20 + j\omega)}{j\omega} \]
\[ = -j20(1 + j\omega/4)\frac{20(1 + j\omega/20)}{\omega} = -j400(1 + j\omega/4)(1 + j\omega/20). \]

- Constant term 400 \( \rightarrow \) 52 dB
- Pole @ origin
- Simple zero with $\omega_c = 4 \text{ rad/s}$
- Simple zero with $\omega_c = 20 \text{ rad/s}$

\[ H(\omega) = \frac{30(10 + j\omega)}{(200 + j2\omega)(1000 + j2\omega)} = \frac{300(1 + j\omega/10)}{200 \times 1000(1 + j\omega/100)(1 + j\omega/500)} \]
\[ = \frac{1.5 \times 10^{-3}(1 + j\omega/10)}{(1 + j\omega/100)(1 + j\omega/500)} \]
- Constant term $1.5 \times 10^{-3}$  $\Rightarrow$  $-56.5$ dB
- Simple zero with $\omega_c = 10$ rad/s
- Simple pole with $\omega_c = 100$ rad/s
- Simple pole with $\omega_c = 500$ rad/s

\[
(f) \quad H(\omega) = \frac{j100\omega}{(100 + j5\omega)(100 + j\omega)^2} = \frac{j10^{-4}\omega}{(1 + j\omega/20)(1 + j\omega/100)^2}
\]

- Constant term $10^{-4}$  $\Rightarrow$  $-80$ dB
- Zero @ origin
• Simple pole with $\omega_c = 20 \text{ rad/s}$
• Simple pole with $\omega_c = 100 \text{ rad/s}$, of order 2
\[ (g) \ H(\omega) = \frac{(200 + j2\omega)}{(50 + j5\omega)(1000 + j\omega)} = \frac{(1 + j\omega/100)}{250(1 + j\omega/10)(1 + j\omega/1000)} \]

- Constant term \(1/250 \implies -48 \text{ dB}\)
- Simple pole with \(\omega_c = 10 \text{ rad/s}\)
- Simple zero with \(\omega_c = 100 \text{ rad/s}\)
- Simple pole with \(\omega_c = 1000 \text{ rad/s}\)
**Problem 9.16** Determine the voltage transfer function $H(\omega)$ corresponding to the Bode magnitude plot shown in Fig. P9.16. The phase of $H(\omega)$ is $90^\circ$ at $\omega = 0$.

![Bode magnitude plot](image)

**Figure P9.16:** Bode magnitude plot for Problem 9.16.

**Solution:** $H(\omega)$ consists of:

1. A constant term $K$ whose dB value is 60 dB, or
   
   $$K = 10^{60/20} = 1000.$$

2. A simple pole of order 3 with $\omega_c = 5$ rad/s (slope $= -60$ dB/decade)

3. A simple zero of order 6 with $\omega_c = 50$ rad/s (slope reverses from $-60$ dB/decade to $+60$ dB/decade)

4. A simple pole of order 3 with $\omega_c = 500$ rad/s (slope changes to 0 dB at $\omega_c = 500$ rad/s).

Hence,

$$H(\omega) = \frac{(j)^N 1000(1 + j\omega/50)^6}{(1 + j\omega/5)^3(1 + j\omega/500)^3} = \frac{j1000(50 + j\omega)^6}{(5 + j\omega)^3(500 + j\omega)^3}.$$

Given that the phase of $H(\omega)$ is $90^\circ$ at $\omega = 0$, it follows that $N = 1$. 
Problem 9.26 For the circuit shown in Fig. P9.26:
(a) Obtain an expression for \( H(\omega) = \frac{V_o}{V_i} \) in standard form.
(b) Generate spectral plots for the magnitude and phase of \( H(\omega) \), given that \( R_1 = 1 \, \Omega \), \( R_2 = 2 \, \Omega \), \( L_1 = 1 \, \text{mH} \), and \( L_2 = 2 \, \text{mH} \).
(c) Determine the cutoff frequency \( \omega_c \) and the slope of the magnitude (in dB) when \( \omega / \omega_c \ll 1 \) and when \( \omega / \omega_c \gg 1 \).

![Figure P9.26: Circuit for Problem 9.26.](image)

Solution:
(a) At node \( V_1 \), KCL gives:
\[
\frac{V_1 - V_i}{R_1} + \frac{V_1}{j\omega L_1} + \frac{V_1}{R_2 + j\omega L_2} = 0.
\]
Also, voltage division gives
\[
V_o = \frac{R_2 V_1}{R_2 + j\omega L_2}.
\]
Solution is
\[
H(\omega) = \frac{V_o}{V_i} = \frac{j\omega L_1 R_2}{(R_1 + j\omega L_1)(R_2 + j\omega L_2) + j\omega L_1 R_1}.
\]
To express denominator in standard form, we factor out \( R_1 R_2 \) and expand terms:
\[
H(\omega) = \frac{j\omega L_1 / R_1}{1 + j\omega \left( \frac{L_1}{R_1} + \frac{L_2}{R_2} + \frac{L_1}{R_2} \right) + j\omega \sqrt{\frac{L_1 L_2}{R_1 R_2}}} = \frac{j\omega K}{1 + j2\xi \omega / \omega_c + (j\omega / \omega_c)^2},
\]
where
\[
K = \frac{L_1}{R_1}, \quad \omega_c = \sqrt{\frac{R_1 R_2}{L_1 L_2}}, \quad \xi = \frac{\omega_c}{2} \left[ \frac{L_1}{R_1} + \frac{L_2}{R_2} + \frac{L_1}{R_2} \right].
\]
(b) For \( R_1 = 1 \, \Omega \), \( R_2 = 2 \, \Omega \), \( L_1 = 1 \, \text{mH} \), and \( L_2 = 2 \, \text{mH} \),
\[
K = 10^{-3}, \quad \omega_c = 10^3 \, \text{rad/s}, \quad \xi = 1.25.
\]
Hence,

$$H(\omega) = \frac{j \times 10^{-3} \omega}{1 + j 2.5 \omega / \omega_c + (j \omega / \omega_c)^2},$$

with $\omega_c = 10^3$ rad/s.

$$M \text{ [dB]} = 20 \log|H(\omega)|$$

Spectral plots of $M \text{ [dB]}$ are $\phi(\omega)$ are shown in Figs. P9.26(a) and (b).

(c) Low-frequency asymptote ($\omega / \omega_c \ll 1$):

$$H(\omega) \simeq j 10^{-3} \omega \implies M \text{ [dB]} \text{ has slope of } +20 \text{ dB/decade.}$$

High-frequency asymptote ($\omega / \omega_c \gg 1$):

$$H(\omega) \simeq - j 10^{-3} \frac{\omega_c^2}{\omega} = - j \frac{10^3}{\omega} \implies M \text{ [dB]} \text{ has slope of } -20 \text{ dB/decade.}$$
2) According to Bode Plot given, we will see that the system shall have
a) one zero at \( \omega_1 = 0 \), one pole at \( \omega_2 = 2\pi \times 10^2 \) and another pole at \( \omega_3 = 2\pi \times 10^4 \)

Thus the transfer function would be like

\[
H(s) = \frac{Ks}{\left(\frac{s}{\omega_2} + 1\right) \left(\frac{s}{\omega_3} + 1\right)}
\]

\[
H(j\omega) = \frac{Kj\omega}{\left(\frac{j\omega}{\omega_2} + 1\right) \left(\frac{j\omega}{\omega_3} + 1\right)} = \frac{Kj\omega}{\left(\frac{\omega}{2\pi} \times 10^2 + 1\right) \left(\frac{\omega}{2\pi} \times 10^4 + 1\right)}
\]

From the Bode plot we notice that at \( \omega = 10^3 \) \( |H(j\omega)| = 0 \text{dB} = 1 \) Thus

\[
|H(j\omega)| = \frac{Kx\omega}{\sqrt{1 + \frac{\omega^2}{\omega_2^2}} \cdot \sqrt{1 + \frac{\omega^2}{\omega_3^2}}} = \frac{K \times 2\pi \times 10^3}{\sqrt{1 + 10^2} \times \sqrt{1 + 0.1^2}}
\]

\[1 \approx \frac{K \times \frac{2\pi \times 10^3}{10 \times 1} \implies K \approx \frac{1}{2\pi \times 10^2}\]

\[
H(s) = \frac{S}{2\pi \times 10^2 \left(\frac{S}{2\pi \times 10^2} + 1\right) \left(\frac{S}{2\pi \times 10^4} + 1\right)}
\]

b1) \( |H(j\omega)|_{\text{dB}} = 20 \left( -\log \left(\frac{2\pi \times 10^2}{\omega}\right) + \log(1) - \log \left| \frac{j\omega}{2000} \right| - \log \left| 1 + \frac{j\omega}{2000} \right| \right)\)

\( \omega_1 = 2\pi \times 10^2 \), \( \omega_2 = 2\pi \times 10^4 \)

\[
|H(j\omega_1)|_{\text{dB}} = 20 \left( -\log \left(\frac{2\pi \times 10^2}{2\pi \times 10^2}\right) - \log \left| 1 + \frac{j2\pi \times 10^2}{2000} \right| - \log \left| 1 + \frac{j2\pi \times 10^4}{2000} \right| \right) = 20 (0 - 0.15051 - 0.00002) = 20 (0.15053) = -3.01 \text{dB}
\]

\[
|H(j\omega_2)|_{\text{dB}} = 20 \left( -\log \left(\frac{2\pi \times 10^4}{2\pi \times 10^2}\right) - \log \left| 1 + \frac{j2\pi \times 10^4}{2000} \right| - \log \left| 1 + \frac{j2\pi \times 10^4}{2000} \right| \right) = 20 (2 - 2.00002 - 0.15051) = -3.01 \text{dB}
\]
\( \Phi(w_1) = 45^\circ \)
\( \Phi(w_2) = -45^\circ \)

\[
H(j\omega) = \frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{(R_2 || \frac{1}{j\omega C_2})}{R_1 + \frac{1}{j\omega C_1}} = -\frac{R_2}{1 + j\omega C_2 \cdot R_2} \left( \frac{1}{j\omega C_1} \right)
\]

Thus the corner freq will be \( f_c_1 = \frac{1}{2\pi R_1 C_1} \) and \( f_c_2 = \frac{1}{2\pi R_2 C_2} \)

It is easy to scale capacitor and resistor value to move the corner freq but we notice that the input impedance is the impedance will be seen from \( V_{in} \) when \( V_{out} \) support no load which is equal to \( Z_1 \) in this case and \( Z_{out} \) will be the impedance the output see when \( V_{in} = 0 \) which in this case is equal to \( Z_2 \). Thus if the input and output impedance are unchanged, so does \( H(j\omega) \). Hence it is impossible to move the corner frequency without changing input/ouput impedan
3a) \[ z_1 = z_2 = 0 \quad P_1 = -0.1 + j \quad P_2 = -0.1 - j \]
\[
H(s) = \frac{s^2}{(s-(0.1+j))(s-(0.1-j))} = \frac{s^2}{(s+0.1-j)(s+0.1+j)}
\]
\[
H(s) = \frac{s^2}{s^2 + 0.2s + 1.01}
\]
\[
H(j\omega) = \frac{(j\omega)^2}{(j\omega-(0.1+j))(j\omega-(0.1-j))} = \frac{-\omega^2}{1.01 + j0.2\omega - \omega^2}
\]
\[
= \frac{\omega^2}{\omega^2 - j0.2\omega - 1.01}
\]

b) The bode plot of this system would be as demonstrated in the Figure 3.2.

That can be explained from Fig 3.1 which \( d_{z_1} \), \( d_{P_1} \), \( d_{P_2} \) demonstrate distance of \( j\omega \) from \( z_1 = 0 \), \( P_1 = -0.1 + j \), \( P_2 = -0.1 - j \). Thus
\[
|H(j\omega)| = \frac{|j\omega - z_1|^2}{|j\omega - P_1||j\omega - P_2|} = \frac{dz_1^2}{dP_1 \cdot dP_2} = \frac{dz_1^2}{dP_1 \cdot dP_2}
\]

when \( \omega = 0 \) so does \( dz_1 \) and \( |H(j\omega)| \) but as \( \omega \) increases the nominator also increase and \( \frac{1}{dP_1} \) but \( \frac{1}{dP_2} \) will decreased. At \( \omega \) close to \( \omega_m \) \( \frac{1}{dP_1} = \frac{1}{0.1} \) whereas \( \frac{1}{dP_2} \) has only dropped from \( \frac{1}{dP_2} @ \omega = 0 \) to \( \frac{1}{dP_2} @ \omega = \omega_m \). Thus created a bump in \( |H| \) value. when \( \omega \) approach infinity \( d_{z_1}, d_{P_1}, d_{P_2} \) will all become approximately the same. Thus limit \( |H(j\omega)| \) approaches one. So the system will pass all high frequency and represent a high-pass filter with a bump at \( \omega_m \)