**Problem 9.36**  Design an active lowpass filter with a gain of 4, a corner frequency of 1 kHz, and a gain roll-off rate of $-60$ dB/decade.

**Solution:** The roll-off rate of $-60$ dB requires a three-stage LP filter, similar in design to that in Fig. 9-26. To secure positive gain, we need an additional fourth stage. Arbitrarily, we choose all resistors of the first three stages to be 10-kΩ resistors, and we realize the overall gain through the last stage.

![Figure P9.36(a)](image)

The transfer function is given by

$$H(\omega) = \frac{V_o}{V_s} = 4 \left( \frac{1}{1 + j\omega/\omega_{c1}} \right)^3,$$

with

$$\omega_{c1} = \frac{1}{R_fC_f}.$$

The problem states that the corner frequency of the overall filter, which we will call $\omega_{c3}$, should be

$$\omega_{c3} = 2\pi f_c = 2\pi \times 10^3 \text{ rad/s}.$$

According to Exercise 9.14,

$$\omega_{c3} = 0.51\omega_{c1}.$$

Hence,

$$\omega_{c1} = \frac{\omega_{c3}}{0.51} = \frac{2\pi \times 10^3}{0.51} = 12.32 \text{ krad/s},$$

and since $R_f = 10 \text{k}\Omega$,

$$C_f = \frac{1}{R_f\omega_{c1}} = 8.12 \text{ nF}.$$

The spectral response of the magnitude of $H(\omega)$ is shown in Fig. P9.36(b).
Figure P9.36(b)
Problem 9.37  Design an active highpass filter with a gain of 10, a corner frequency of 2 kHz, and a gain roll-off rate of 40 dB/decade.

Solution: To secure a roll-off rate of 40 dB/decade we need to use two stages of the circuit in Fig. 9-24.

![Circuit Diagram](image)

The two stages have the same input impedances ($R_s$ and $C_s$). We choose

$$R_{f1} = R_s = 10 \, \text{k}\Omega, \quad R_{f2} = 100 \, \text{k}\Omega.$$

Consequently,

$$G_1 = -\frac{R_{f1}}{R_s} = -1, \quad G_2 = -\frac{R_{f2}}{R_s} = -10.$$

The overall response is:

$$H(\omega) = \frac{V_o}{V_s} = G_1G_2 \left( \frac{j\omega/\omega_{HP}}{1 + j\omega/\omega_{HP}} \right)^2$$

$$= 10 \left( \frac{j\omega/\omega_{HP}}{1 + j\omega/\omega_{HP}} \right)^2,$$

with

$$\omega_{HP} = \frac{1}{R_sC_s}.$$

The problem statement specifies a corner frequency $f_c = 2$ kHz with a corresponding angular frequency $\omega_c$ given by

$$\omega_c = 2\pi f_c = 4\pi \times 10^3 \, \text{rad/s}.$$

By definition, $\omega_c$ is the angular frequency at which the magnitude of $H(\omega)$ is equal to 0.707 of its maximum value. Thus, at $\omega = \omega_c$,

$$|H(\omega_c)| = 10 \left| \left( \frac{j\omega_c/\omega_{HP}}{1 + j\omega_c/\omega_{HP}} \right)^2 \right| = 7.07,$$

which leads to

$$\frac{x^2}{1 + x^2} = 0.707$$

with $x = \omega_c/\omega_{HP}$.

Solution of the above equation gives

$$x = 1.55.$$
Hence

\[ \omega_{HP} = \frac{1}{R_s C_s} = 1.55 \omega_c = 1.55 \times 4\pi \times 10^3 = 1.95 \times 10^4 \text{ rad/s}, \]

and

\[ C_s = \frac{1}{R_s \omega_{HP}} = \frac{1}{10^4 \times 1.95 \times 10^4} = 5.1 \times 10^{-9} \, \text{F} = 5.1 \, \text{nF}. \]

A plot of \( M \, [\text{dB}] \) is shown in Fig. P9.37(b).
Problem 9.38  The element values in the circuit of the second-order bandpass filter shown in Fig. P9.38 are: $R_{f1} = 100 \, \text{k\Omega}$, $R_{s1} = 10 \, \text{k\Omega}$, $R_{f2} = 100 \, \text{k\Omega}$, $R_{s2} = 10 \, \text{k\Omega}$, $C_{f1} = 3.98 \times 10^{-11} \, \text{F}$, $C_{s2} = 7.96 \times 10^{-11} \, \text{F}$. Generate a spectral plot for the magnitude of $H(\omega) = V_0/V_s$. Determine the frequency locations of the maximum value of $M \, [\text{dB}]$ and its half-power points.

Solution: The overall transfer function is given by

$$H(\omega) = \frac{V_{\text{out}}}{V_s} = G_{\text{LP}}^2 G_{\text{HP}}^2 \left( \frac{1}{1 + j\omega/\omega_{\text{LP}}} \right)^2 \left( \frac{j\omega/\omega_{\text{HP}}}{1 + j\omega/\omega_{\text{HP}}} \right)^2,$$

with

$$G_{\text{LP}} = -\frac{R_{f1}}{R_{s1}} = -10,$$

$$G_{\text{HP}} = -\frac{R_{f2}}{R_{s2}} = -10,$$

$$\omega_{\text{LP}} = \frac{1}{R_{f1} C_{f1}} = \frac{1}{10^5 \times 3.98 \times 10^{-11}} = 251.26 \, \text{krad/s},$$

$$\omega_{\text{HP}} = \frac{1}{R_{s2} C_{s2}} = \frac{1}{10^4 \times 7.96 \times 10^{-11}} = 125.63 \, \text{krad/s}.$$
Figure P9.38(b)

From the plot we determine that:

\[ \omega_0 = 178 \text{ krad/s} \quad (M(\omega_0) = 72.9563 \text{ dB}), \]
\[ \omega_1 (-3 \text{ dB}) = 94 \text{ krad/s}, \]
\[ \omega_2 (-3 \text{ dB}) = 336 \text{ krad/s}. \]
2. (a) The given system $H(s)$, has:

- Zeros: $f_{z_1} = 10 \, [Hz]$, $f_{z_2} = 10^7 \, [Hz]$
- Poles: $f_{p_1} = 10^3 \, [Hz]$, $f_{p_2} = 10^5 \, [Hz]$

The slopes are $-20 \, [dB/dec]$

![Graph showing the magnitude response of $H(s)$](image)

The actual values are calculated from the following formula –

$$|H(s)| = 20 \cdot \log\left(\frac{\omega^2}{4 \pi^2 \cdot 10^2 + 1}\right) + \log\left(\frac{\omega^2}{4 \pi^2 \cdot 10^4 + 1}\right) - \log\left(\frac{\omega^2}{4 \pi^2 \cdot 10^6 + 1}\right) - \log\left(\frac{\omega^2}{4 \pi^2 \cdot 10^{10} + 1}\right) =$$

$$20 \cdot \log\left(\sqrt{\frac{f^2}{10^2} + 1}\right) + \log\left(\sqrt{\frac{f^2}{10^4} + 1}\right) - \log\left(\sqrt{\frac{f^2}{10^6} + 1}\right) - \log\left(\sqrt{\frac{f^2}{10^{10} + 1}}\right)$$

Values can be determined by plugging the frequency into the above formula, e.g. (marked on actual plot below):

$$|H(s = j2 \pi \cdot 1 MHz)| = 20 \cdot \log\left(\sqrt{\frac{10^{12}}{10^2} + 1}\right) + \log\left(\sqrt{\frac{10^{12}}{10^4} + 1}\right) - \log\left(\sqrt{\frac{10^{12}}{10^6} + 1}\right) - \log\left(\sqrt{\frac{10^{12}}{10^{10} + 1}}\right) =$$

$$100[dB] + 0.04[dB] - 60[dB] - 20.04[dB] = 20[dB]$$
(b) \[ |H(s = j2\pi \cdot 10Hz)| = 3.01 [dB] \]

\[ |H(s = j2\pi \cdot 10KHz)| = 39.91 [dB] \]

\[ |H(s = j2\pi \cdot 10MHz)| = 3.01 [dB] \]

3. (a) Notice that the given input impedance is real, therefore it either consists of resistors only, or resistors in addition to a combination of capacitors and inductors which cancel each other at the center frequency.

\[ R' = K_m \cdot R \Rightarrow K_m = \frac{20K\Omega}{1K\Omega} = 20 \]

The center frequency is shifted, therefore –

\[ \omega' = K_f \cdot \omega \Rightarrow K_f = \frac{100 KHz}{5 KHz} = 20 \]
The net scaling factors for the components of the circuit are as following –

\[ R' = K_m \cdot R = 20 \cdot R \]

\[ C' = \frac{1}{K_m \cdot K_f} \cdot C = \frac{1}{400} \cdot C \]

\[ L' = \frac{K_m}{K_f} \cdot L = L \]

In order for the quality factor to remain the same we require –

\[ Q' = \frac{\omega_0'}{B'} = \frac{\omega_b'}{B'} \Rightarrow \frac{2\pi \cdot 100kHz}{B'} = \frac{2\pi \cdot 5kHz}{500Hz} \]

\[ \Rightarrow B' = 10kHz \]

\[ \Rightarrow \omega_1' = 95kHz, \ \omega_2' = 105kHz \]

These values may be applied to different circuits, e.g. the example shown on page 441 in the book or others band-pass filters.

(b) Op-amp should be able to operate on high frequencies.

Op-amp should be as ideal as possible (High input resistance, low output resistance, high gain).

4. The general form of a transfer function with two zeros and two poles –

\[ H(s) = \frac{K \cdot (s-s_{z_1})(s-s_{z_2})}{(s-s_{p_1})(s-s_{p_2})} \]

In the given problem we have the following –

- Zeros: \( s_{z_1} = 0, \ s_{z_2} = -3 \)
- Poles: \( s_{p_1} = -1+4j, \ s_{p_2} = -1-4j \)

Therefore –

\[ H(s) = \frac{K \cdot (s-0)(s+3)}{(s-(-1+4j))(s-(-1-4j))} = \frac{K \cdot s \cdot (s+3)}{(s-(-1+4j))(s-(-1-4j))} \]

Applying the given assumption –

\[ \lim_{\|s\| \to \infty} \frac{K \cdot s \cdot s}{s \cdot s} = 1 \Rightarrow K = 1 \]

\[ \Rightarrow H(s) = \frac{s \cdot (s+3)}{(s-(-1+4j))(s-(-1-4j))} \]