Magnetoresistance of a YBa$_2$Cu$_3$O$_7$ Corbino disk: Probing geometrical contributions to the unconventional normal-state magnetoresistance of high-temperature superconductors

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Complementary measurements of geometrical effects in the normal-state magnetoresistance of YBa$_2$Cu$_3$O$_7$ thin films, patterned into a Corbino disk, a square, and a strip-shaped sample, are reported. We observe a difference between the Corbino and strip-sample magnetoresistances, which arises from bending of the current trajectories in the transverse magnetic field and equals the square of the Hall angle in YBa$_2$Cu$_3$O$_7$. This geometrical magnetoresistance component vanishes when the magnetic field is oriented parallel to the film. The geometrical magnetoresistance has also been found to be significant in square-shaped samples, and thus, has to be considered in magnetoresistance measurements of YBa$_2$Cu$_3$O$_7$ single crystals.

The unconventional transport properties in the normal state of a high-temperature superconductor (HTS) have attracted much interest over the past few years from both the experimental and theoretical points of view. In particular, the temperature variations of the Hall effect and the magnetoresistance (MR) could not have been explained in terms of the Fermi-liquid transport theory, and the two distinct scattering times $\tau_0$ and $\tau_H$, governing the carrier motion parallel to the applied electric field and to the cyclotronic trajectories, respectively, had to be introduced. In a conventional one-band model, both the Hall angle $\tan \theta_H \approx \omega_c \tau_H$, where $\omega_c$ is the cyclotron frequency, and the orbital MR $\Delta \rho / \rho_0 = [\rho(B) - \rho(0)] / \rho(0) \approx (\omega_c \tau_H)^2$ are associated with the relaxation time $\tau_H$. Thus, it might be expected that $\Delta \rho / \rho_0 \approx (\tan \theta_H)^2$ holds at temperatures considerably above $T_c$, where the superconducting fluctuation effects do not contribute significantly to both quantities. Since

$$\tan \theta_H = (AT^2 + C)^{-1}$$

applies to most HTS’s, where $A$ and $C$ are constants (with $C \approx 0$ for optimally doped HTS), the unconventional MR closely follows the $T^{-4}$ temperature dependence. Such behavior was verified in 60-K and 90-K YBa$_2$Cu$_3$O$_7$, optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$, overdoped Tl$_2$Ba$_2$CuO$_y$, and optimally doped La$_{2-x}$Sr$_x$CuO$_4$ but recently challenged in underdoped La$_{2-x}$Sr$_x$CuO$_4$. In this paper, we discuss the fundamental aspects of the MR temperature dependence measurements apart from the obvious fact that in the normal state $\Delta \rho / \rho_0$ is very small in HTS and experimental errors are likely to cause significant discrepancies.

To accurately measure the “physical” MR (PHMR), i.e., the change of the diagonal components of the material’s resistivity tensor, the current path must remain unchanged in the magnetic field. This prerequisite can be enforced by a suitable sample geometry, e.g., in the straight-line, striplike sample with large length-to-width ratio $n = l/w \to \infty$. On the contrary, bending of the carrier’s trajectories is maximized in the Corbino disk geometry, where the current is injected between the center and the rim of a flat disk in a transverse (axial) magnetic field. In fact, the Corbino arrangement directly probes the change of the sample’s conductivity $\sigma$ in a magnetic field. The significant difference of the two geometries can be easily understood within the simple Drude model, which states that the resistance changes in a magnetic field are $R(B)/R(0) = \rho(B)/\rho(0) = 1$ for long wires and $R(B)/R(0) = \sigma(0)/\sigma(B) = 1 + (\omega_c \tau_H)^2$ for the Corbino disk, respectively. Of course, both results are physically equivalent if proper inversion from magnetoresistivity to magnetoconductivity tensors is considered. In real materials, the Drude model does not apply exactly and, thus, $R(B)/R(0) \neq 1$, but nevertheless the MR of strips is significantly different from that of the Corbino disk. This difference is called the “geometrical” MR (GEMR) since it results from bending of the current trajectories in a magnetic field and is closely related to the value of the Hall angle in the material. For transport measurements, the infinitely long strip and the Corbino disk are opposing topological shapes, and any practical, rectangular-shaped samples of limited dimensions exhibit some GEMR contribution. This latter condition is most important for HTS single-crystal MR measurements, since the sample shapes are usually close to a square ($n = 1$) and a substantial GEMR is expected, obscuring the PHMR.

In this paper, we compare results obtained on a Corbino disk ($n = 1$), a square ($n = 1$), and a strip-shaped sample ($n = 11$) fabricated in the very same YBa$_2$Cu$_3$O$_7$ (YBCO) thin film. The aim of this work is twofold: first, we demonstrate that Corbino MR exhibits a similar unconventional temperature dependence to that observed in single crystals, and, second, we investigate a possible contamination of recently published MR results by the GEMR in square samples.
Theoretically, only a sample with \( n \rightarrow \infty \) has zero GEMR and, thus, allows for a direct measurement of PHMR \( \Delta \rho^{\text{phys}} / \rho_0 = (\rho_{xx}(B) - \rho_{xx}(0))/(\rho_{xx}(0)) \). On the other hand, the Corbino disk arrangement \((n=0)\) maximizes GEMR. In the general case of an arbitrarily shaped sample with a small Hall angle \( \theta_H \), the total MR equals
\[
\Delta R / R_0 = \Delta \rho^{\text{phys}} / \rho_0 + \Delta \rho^{\text{geom}} / \rho_0, \tag{2}
\]
where GEMR is given by
\[
\frac{\Delta \rho^{\text{geom}}}{\rho_0} = \left(1 + \frac{\Delta \rho^{\text{phys}}}{\rho_0}\right) \tan^2 \theta_H g(n) \tag{3}\]
for rectangular samples with perfectly conducting current contacts on opposite faces. The geometric factor \( g(n) \) can be approximated\(^9\) for \( n > 1 \) by \( g(n) = 0.543/n \) and for \( n < 1 \) by \( g(n) = 1 - 0.543n \). For the Corbino disk \( g(0) = 1 \), and for square samples \( g(1) = 0.5 \).

In HTS materials, far above \( T_c \), the in-plane MR \( \Delta \rho / \rho_0 \ll 1 \) (Refs. 3–7), and thus, the second term in the bracket in Eq. (3) can be neglected. Hence, the GEMR term is approximately equal to
\[
\Delta \rho^{\text{geom}} / \rho_0 \approx \tan^2 \theta_H g(n). \tag{4}
\]
We stress again that the importance of GEMR, compared to PHMR, depends primarily on the sample shape. In the Fermi-liquid transport theory, \( \tan^2 \theta_H \) and the magnitude of PHMR are related. Thus, a small \( \theta_H \), which implies a reduced GEMR, is commonly associated with small PHMR. The GEMR, like the PHMR, is proportional to \( B^2 \) in moderate magnetic fields and cannot be separated from the latter experimentally. If an investigated material cannot be fabricated into a long strip, only an additional measurement of the Hall angle allows, according to Eqs. (2) and (3), for an evaluation of the ‘true MR,’ i.e., the PHMR.

For a complementary investigation of PHMR and GEMR, three independent thin-film structures were prepared in the same YBCO film, namely, a conventional long-strip geometry with six voltage probes \((n=11)\), a square sample, and a Corbino disk. The 100\(\pm\)10-nm-thick, epitaxial YBCO films were deposited by single-target rf sputtering on LaAlO\(_3\) substrates. The films exhibited a 0.5-K-wide superconducting transition at 90 K, and their critical current density exceeded 3000 A/cm\(^2\) at 77 K. The test structures were patterned into the film, using our laser processing method.\(^{10}\) Laser patterning in nitrogen atmosphere, based on a local, heat-induced decomposition of YBCO into nonconducting phases, allowed us to obtain a very precise (1-\(\mu\)m-resolution) definition of the structure geometry and good electrical insulation of the structure from the rest of the film with no noticeable deterioration of the YBCO properties. The strip sample was 0.4-mm wide with a voltage probe distance of 4.4 mm. In case of a square sample, the area between the voltage probes—Au wires crossing the entire sample—was \(1.6 \times 1.6\) mm\(^2\). The Corbino structure had the shape of a ring with 4-mm outer diameter and an electrically inactive area of a 2-mm diameter in the center that was used for contacts. Ag contact pads were vacuum deposited and used to connect Au wires with a Ag paste.

The traditional Corbino measurement is a two-probe method that employs contacts at the center and around the rim of the disk. The contact resistance and its possible magnetic-field dependence in this configuration might introduce considerable ambiguities. Thus, we used a modified geometry, shown in Fig. 1, that utilized pairs of separate current and voltage contacts at the center and the rim of the disk, respectively. Such an arrangement, although not exactly equivalent to the traditional Corbino disk, which requires an entirely circumscribing electrode of perfect conductivity,\(^{11}\) provided a similar short circuit for the Hall voltage. The equivalence of the two structures in materials with small Hall angle, like the HTS, was independently checked by us using 0.1 \(\Omega\) cm \(p\)-type Si samples. The MR of the modified structure agreed with the conventional arrangement within the experimental accuracy of the measurement.

The MR and the Hall effect were measured in a closed-cycle refrigerator using a temperature controller equipped with two platinum thermometers. The magnetic field of 1 T was provided by an electromagnet, and each data point was taken at both field polarities. The sample’s temperature was recorded at zero field, and the temperature control was performed by the thermometer mounted outside the electromagnet’s pole caps. By appropriate settings of time constants of the temperature controller, we additionally assured that the magnetoresistance of the platinum thermometer in the stray field did not evoke any systematical temperature changes. Such an effect would lead to an apparent negative MR of our samples. All measurements were carried out using a lock-in technique and highly constant-amplitude ac currents of 17 Hz. The same current was passed through both the strip and the Corbino structures connected in series, and the MR was measured simultaneously in both samples. A fully computer-controlled setup enabled us to acquire up to 1500 data points for a single MR value, thus, extending the resolution of the MR measurement at higher temperatures to better than \(10^{-6}\) at \(B = 1\) T.

Figure 2 shows the resistivity \(\rho_{xx}\) in the zero magnetic field and the Hall coefficient \(R_H\) of our strip sample. The data are typical for a high-quality YBCO film, and the Hall angle follows Eq. (1) with \(A = 0.0428\) K\(^{-2}\) and \(C = 96.2\) up to
about 170 K. At higher temperatures, cot $\theta_H$ versus $T^2$ (not shown) exhibits a slightly sublinear behavior, as observed in many HTS samples.\textsuperscript{12}

Figure 3 compares $\Delta R/R_0$ of the three different sample geometries in a transverse magnetic field of $B=1$ T. We note that the measurements collapse in the vicinity of $T_c$, but with the increase of temperature, the MR of the Corbino disk (MRD) becomes significantly larger than the MR of the strip (MRS). The MR of the square sample is shown for selected temperatures only and is in between the MRD and MRS data as predicted by Eqs. (2) and (3). Close to $T_c$, superconducting fluctuations dominate the MR and tend to reduce the Hall angle,\textsuperscript{13} significantly reducing GEMR. Only at high temperatures, a crossover to the normal-state MR does occur,\textsuperscript{14} as is clearly reflected by the divergence of the MRD and MRS curves. Above 125 K, the MRD follows a power-law behavior and is proportional to $T^{-3.8}$, which is much different from the Corbino MR in conventional materials, and indicates an unconventional behavior of the HTS conductivity change in a transverse magnetic field. The solid line in Fig. 3 represents the prediction for the GEMR in the Corbino disk, calculated from the Hall data obtained on the strip sample according to Eq. (4). A similar calculation using $g(11)=0.05$ reveals that, as expected, the GEMR of our strip sample is more than two orders of magnitude smaller than the PHMR over the entire investigated temperature range and can be safely neglected for this sample geometry. We wish to point out that the difference between the MRD and MRS (triangles in Fig. 3)—the experimental GEMR of the Corbino disk—is in the excellent agreement with the prediction calculated from $\theta_H$ [Eq. (4)].

The longitudinal MR, where the magnetic field is oriented parallel to the current, should be free of orbital MR effects and, in particular, of the GEMR. Figure 4 demonstrates that the MR in this orientation is in fact the same for both the strip and Corbino geometries and is substantially smaller than the transverse MR, reflecting the absence of orbital contributions.

Finally, we want to discuss some implications of our results. First, we note that the calculation of $g(n)$ is based on rectangular samples with contacts on opposite faces. If the voltage probes are prepared by conducting wires across the specimen, as for our square sample, the proper length to determine $g(n)$ is the distance between these wires. In fact, the GEMR of our square sample is about half the value observed for the Corbino disk. As one can note in Fig. 3, this will lead to a significant error in PHMR measurements of HTS single crystals which, in most cases, cannot be prepared as long strips.

MR investigations of HTS single crystals in the normal state\textsuperscript{3} revealed a violation of Kohler’s rule and an unconventional temperature dependence of $\Delta \rho/\rho_0$. It was argued that the normal-state MR probes the variance of the local Hall angle over the Fermi surface and, thus, $\Delta \rho/\rho_0 \propto \tan^2 \theta_H$. However, Eq. (4) also implies the very same temperature dependence for the GEMR, making it impossible to experi-
mentally separate GEMR and PHMR in a single MR-type measurement. Since Eq. (1) is closely obeyed in most of the HTS materials both the PHMR and the GEMR are expected to be proportional to $T^{-4}$ in the normal state (assuming $C \approx 0$). Close inspection of Fig. 3 shows that our MRS data, which are essentially free of GEMR, confirm the unconventional $\Delta \rho_{\text{phys}}/\rho_0 \approx T^{-4.0}$ behavior (dotted line), but simultaneously are not exactly proportional to $\tan^2 \theta_H$ (solid line). On the other hand, $\tan \theta_H$ deviates from the $T^{-2}$ behavior at high temperatures and, hence, neither $\tan^2 \theta_H$ nor the MRD exhibit the $T^{-4}$ variation. We conclude that $(\Delta \rho_{\text{phys}}/\rho_0)/\tan^2 \theta_H$ is not constant for our strip sample, contradicting previous claims; instead it approaches almost a constant value in the MRD, due to the substantial GEMR contribution. We attribute the observed discrepancy between the temperature dependences of PHMR and $\tan^2 \theta_H$ not as a failure of the theoretical predictions by Harris et al., but due to a still-noticeable contribution of the MR fluctuation term, which has been shown to extend to at least $2T_c$.

In summary, we have presented complementary measurements of MR in several geometries and found that the GEMR in YBCO is proportional to the square of the Hall angle. The GEMR may reach the same magnitude as the PHMR near room temperature in square samples—a typical geometry for single-crystal HTS measurements. The unconventional temperature dependence of the PHMR in YBCO is confirmed, but it does not follow exactly the square of the Hall angle, which we believe is due to the contribution of superconducting fluctuations even at elevated $T \approx T_c$ temperatures.

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1For a review, see N. P. Ong et al., in *High Tc Superconductivity and the C60 Family*, edited by S. Feng and H. C. Ren (Gordon and Breach, Newark, NJ, 1995), p. 53.
7F. F. Balakirev et al., cond-mat/9705107 (unpublished).